Equidistant sets and convex bodies

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The equidistant set of two nonempty subsets K and L (called focal sets) in the Euclidean space $\mathbb{R}^n$ is the set of all points that have the same distance from both K and L, measured with the infimum metric. The most well-known examples of equidistant sets in the plane (apart from segment and angle bisectors) are the classical conics, which means that we can consider equidistant sets as a kind of their generalizations.

But the class of equidistant sets is much wider than this. In [1], Vincze proved that all closed convex planar curves in the plane are equidistant sets, by describing a method to generate the two focal sets for any closed convex polygon, and approximating general closed convex planar curves by these polygons and using the continuity theorem of Ponce and Santibanez (see [2]). It turns out that this result is not bound to dimension two: we prove that in any dimensions, the boundary of any convex polytope can be generated as an equidistant set, and by applying the approximation technique from above, the same is true for any convex body (see [3]). Thus, in some sense, equidistancy is a generalization of the notion of convexity.

By this method, we can generate any convex polytope as the equidistant set of an inner singleton and an outer focal set having as many elements as the number of the facets. Introducing the notion of equidistant polytopes, classified by the cardinalities of their focal sets, we can say that all convex polytopes having m facets are equidistant polytopes of type $(1, m)$. Stepping out of the discrete case, the authors of [2] posed the question to characterize all closed sets that can be realized as the equidistant set of two *connected* disjoint closed sets. We show that by connecting the points in the outer focal set in an adequate way, we can achieve such a focal set for any convex polytope.

References

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