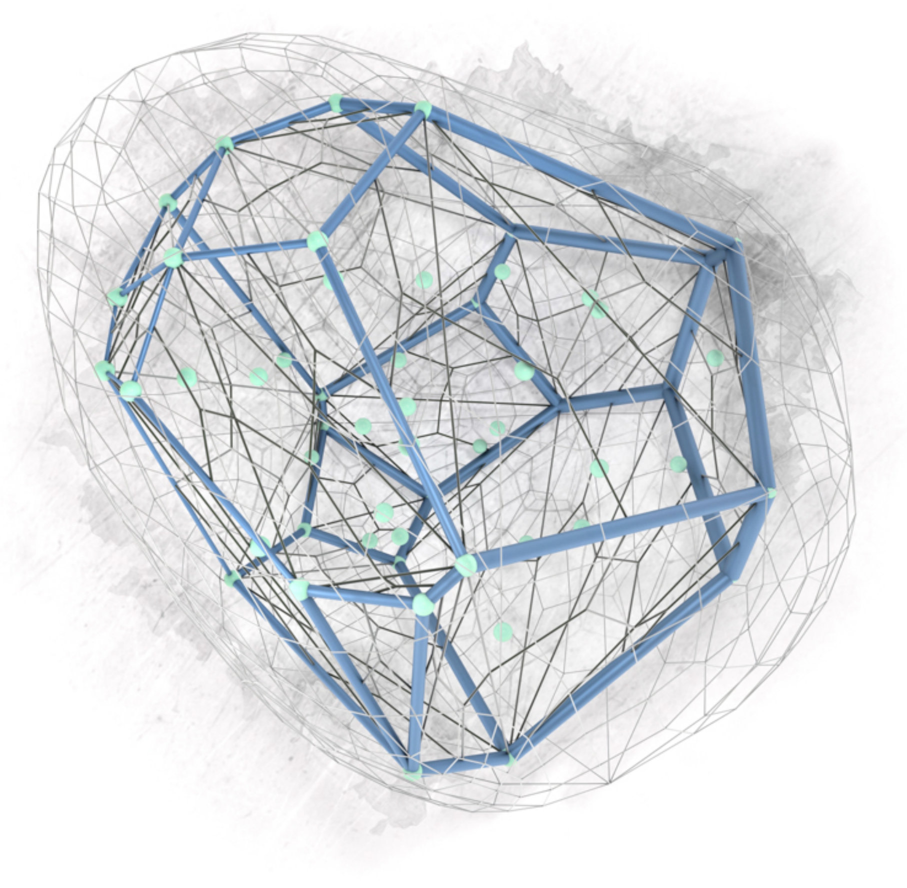
2022 Szeged Workshop on Convexity

27-28 May, 2022

University of Szeged, Hungary



Book of Abstracts

Programme

May 27, Friday

8:30 – 8:55	*Registration*
9:00 – 9:50	**Eva Kopecká**, *Projecting points between convex sets*
10:00 – 10:25	**Imre Bárány**, *The cocked hat*
10:30 – 11:00	*Coffee break*
11:00 – 11:50	**Márton Naszódi**, *Löwner's problem for log-concave functions*
12:00 – 12:25	**Grigory Ivanov**, *No-dimension versions of classical theorems of combinatorial convexity in Banach spaces*
12:30 – 14:00	*Lunch break*
14:00 – 14:50	**Anna Gusakova**, *Spherical convex hull of random points on a wedge*
15:00 – 15:25	**Florian Besau**, *Weighted floating body of polytopes*
15:30 – 16:00	*Coffee break*
16:00 – 16:25	**Konrad Swanepoel**, *Totally separable translative packings of a convex body*
16:30 – 16:55	**Károly Bezdek – Zsolt Lángi**, *From the separable Tammes problem to extremal distributions of great circles in the unit sphere*
17:00 – 17:25	**Ji Hoon Chun**, *The sausage conjecture*
17:30 – 18:30	*Poster session*
	Bushra Basit, *Discrete isoperimetric problems in spaces of constant curvature*
	Balázs Grünfelder, *Variance estimates for generalized random polygons*
	Emőke Imre, *The implicit function theorem and convexity*
	Zsolt Lángi, *From the separable Tammes problem to extremal distributions of great circles in the unit sphere, Part II*
	Kinga Nagy, *Best and random approximations with generalized disc-polygons*
	Márk Oláh, *Equidistant sets and convex bodies*
	Dániel István Papvári, *Random approximations by generalized disc-polygons*
	Ádám Sagmeister, *The isodiametric problem in spaces of constant curvature and its stability*
18:30 – 21:00	*Reception*

May 28, Saturday

9:00 – 9:50	**Eugenia Saorín Gómez**, *Remarks on certain parallel linear behavior of mixed volumes and mixed discriminants*
10:00 – 10:25	**Nico Lombardi**, *Inner and outer mean radii of convex bodies*
10:30 – 11:00	*Coffee break*
11:00 – 11:50	**Fabian Mussnig**, *Functional intrinsic volumes*
12:00 – 12:25	**Jonas Knoerr**, *Unitarily invariant valuations on convex functions*
12:30 – 14:00	*Lunch break*
14:00 – 14:25	**Eduardo Lucas**, *On discrete log-Brunn-Minkowski type inequalities*
14:30 – 14:55	**Jesús Yepes Nicolás**, *Mass-distribution of partitions of convex bodies by hyperplanes*
15:00 – 15:25	**Francisco Marín Sola**, *On extensions of Grünbaum's inequality*
15:30 – 15:55	**Mihály Bessenyei**, *Generalized monotonicity in terms of differential inequalities*

Abstracts

The cocked hat

Imre Bárány
Alfréd Rényi Institute of Mathematics, and University College London
(Joint work with William Steiger and Sivan Toledo)

An old question in navigation, the so-called cocked hat problem, has various old solutions. We examine under what conditions these solutions are valid.

Discrete isoperimetric problems in spaces of constant curvature

Bushra Basit
Budapest University of Technology and Economics (Hungary)
(Joint work with Zsolt Lángi)

It was proved by Böröczky and Peyerimhoff that among simplices inscribed in a ball in spherical and hyperbolic space, respectively, the regular simplices have maximal volume. In this lecture we show that among simplices circumscribed about a ball in hyperbolic space, the regular simplices have minimal volume. We also investigate analogous questions for d-dimensional spherical and hyperbolic polytopes with $d+2$ vertices.

Weighted Floating Body of Polytopes

Florian Besau
Technische Universität Wien (Austria)
(Joint work with Carsten Schütt and Elizabeth M. Werner)

In this talk I will present on a work where we establish asymptotic results for the weighted floating body of n-dimensional convex polytopes. The weighted floating body is a generalization of the classical floating body which arises by replacing the usual Euclidean volume with a measure that has a positive and continuous density function.

In our results we establish an interesting connection between the volume of the floating body and the number of complete flags of a polytope. This flag number is an important combinatorial invariant and is the same as the

number of simplices obtained in the barycenter subdivision. The flag number is minimized for the simplex and it is an open conjecture by Kalai, for $n \geq 4$, that the flag number of symmetric convex polytopes is minimized for the n-dimensional cube.

Generalized monotonicity in terms of differential inequalities

Mihály Bessenyei
University of Debrecen (Hungary)

The classical notions of monotonicity and convexity can be characterized via the nonnegativity of the first and the second derivative, respectively. These notions can be extended applying Chebyshev systems. The aim of this note is to characterize generalized monotonicity in terms of differential inequalities, yielding analogous results to the classical derivative tests. Applications in the fields of convexity and differential inequalities are also discussed.

From the separable Tammes problem to extremal distributions of great circles in the unit sphere

Károly Bezdek – Zsolt Lángi
University of Calgary (Canada); Budapest University of Technology and Economics (Hungary)

A family of spherical caps of the 2-dimensional unit sphere $\mathbb{S}^2$ is called a totally separable packing, in short, a TS-packing if any two spherical caps can be separated by a great circle which is disjoint from the interior of each spherical cap in the packing. The separable Tammes problem asks for the largest density of given number of congruent spherical caps forming a TS-packing in $\mathbb{S}^2$. We solve this problem up to 8 spherical caps and upper bound the density of any TS-packing of congruent spherical caps in terms of their angular radius. Based on this, we show that the centered separable kissing number of a 3-dimensional Euclidean ball is 8. Furthermore, we prove bounds for the maximum of the smallest inradius of the cells of the tilings generated by $n > 1$ great circles in $\mathbb{S}^2$. Next, we prove dual bounds for TS-coverings of $\mathbb{S}^2$ by congruent spherical caps. Here a covering of $\mathbb{S}^2$ by spherical caps is called a totally separable covering in short, a TS-covering if there exists a tiling generated by finitely many great circles of $\mathbb{S}^2$ such that the cells of the tiling are covered by pairwise distinct spherical caps of the covering. Finally, we extend some of our bounds on TS-coverings to spherical spaces of dimension > 2.

The Sausage Conjecture

Ji Hoon Chun
Technische Universität Berlin (Germany)
(Joint work with Martin Henk)

The Sausage Conjecture of L. Fejes Tóth (1975) states that for all dimensions $d \geq 5$, the densest packing of any finite number of spheres in $\mathbb{R}^d$ occurs if and only if the sphere centers are all placed as closely as possible on one line, i.e., a "sausage." We discuss the progress made in the literature, including the result of Betke and Henk (1998) that the Sausage Conjecture is true for all $d \geq 42$. Our work builds upon the methods of Betke and Henk to improve the lower bound to $d \geq 36$ with the aid of interval arithmetic for certain complicated portions. We also mention some potential future research directions.

Variance estimates for generalized random polygons

Balázs Grünfelder
University of Szeged (Hungary)
(Joint work with F. Fodor and V. Vígh)

We prove asymptotic lower bounds on the variance of the number of vertices and missed area of random disc-polygons in convex discs whose boundary is C^2_+ smooth. The established lower bounds are of the same order as the upper bounds proved previously in Fodor, Vígh (2018).

We also prove asymptotic upper bounds on the variance of the number of vertices and missed area of random L-convex polygons. For two convex discs K and L, we say that K is L-convex if it is the intersection of all translates of L containing K. We consider two cases: first we assume that the the curvature of K and L are strictly bounded away from each other. In the second case we assume that $K = L$.

Spherical convex hull of random points on a wedge

Anna Gusakova
University of Münster (Germany)
(Joint work with F. Besau, M. Reitzner, C. Schütt, C. Thäle, E. M. Werner)

Consider a convex body $K \subset \mathbb{R}^d$ and let $X_1, \dots, X_n$ be i.i.d. points uniformly distributed in K. Denote by K_n a random polytope, constructed as a convex hull of points $X_1, \dots, X_n$. It is one of the classical models of random polytope, which has been intensively studied during the last years. One of the first natural questions is to determine the asymptotics of the average number of k-faces of K_n as $n \to \infty$. In $\mathbb{R}^d$ this problem is solved (up to certain extend).

In this talk we consider a spherical analogue of the above construction. More precisely we will deal with the special type of spherical convex bodies, which include two antipodal points. More precisely consider two half-spaces H_1^+ and H_2^+ in $\mathbb{R}^{d+1}$ whose bounding hyperplanes H_1 and H_2 are orthogonal and pass through the origin. The intersection $\mathbb{S}^d_{2,+} := \mathbb{S}^d \cap H_1^+ \cap H_2^+$ is a spherical convex subset of the d-dimensional unit sphere $\mathbb{S}^d$, which contains a great subsphere of dimension $d-2$ and is called a spherical wedge. Choose n independent random points uniformly at random on $\mathbb{S}^d_{2,+}$ and consider the expected facet number of the spherical convex hull of these points. We will shown that, up to terms of lower order, this expectation grows like a constant multiple of $\log n$. At the end of the talk we will compare this result to the corresponding behaviour of classical Euclidean random polytopes and of spherical random polytopes on a half-sphere.

The implicit function theorem and convexity

Emőke Imre
Óbuda University, Budapest (Hungary)

Let $\mathbf{f} : \mathbf{R}^{\mathrm{n+m}} \to \mathbf{R}^{\mathrm{m}}$ be a continuously differentiable function. $\mathbf{R}^{\mathrm{n+m}}$ is the direct sum $\mathbf{R}^{\mathrm{n}} + \mathbf{R}^{\mathrm{m}}$, a point of this is $(\ \mathbf{x},\, \mathbf{y}) = (\mathrm{x}_1, \dots, \mathrm{x}_\mathrm{n}, \mathrm{y}_1, \dots, \mathrm{y}_\mathrm{m})$. Starting from the given function $\mathbf{f}$, the goal is to construct a function $\mathbf{g}\mathbf{R}^{\mathrm{n}} \to \mathbf{R}^{\mathrm{m}}$ whose graph $(\mathbf{x}, g(\mathbf{x}))$ is precisely the set of all $(\mathbf{x}, \mathbf{y})$ such that $\mathbf{f}(\mathbf{x}, \mathbf{y}) = \mathbf{0}$.

Fix a point $(\mathbf{a}, \mathbf{b}) = (a_1, \dots, a_n, b_1, \dots, b_m)$ with $\mathbf{f}(\mathbf{a}, \mathbf{b}) = \mathbf{0}$, where $\mathbf{0}$ is the element of $\mathbf{R}^m$. If the matrix $\left[\left(\partial f_{\mathrm{i}} / \partial y_j \right) (\mathbf{a},\, \mathrm{b})\ \right]$ is invertible, then there exists an open set U containing $\mathbf{a}$, an open set V containing $\mathbf{b}$, and a unique function g $U \to V$ such that

$$\{\mathbf{x}, \mathbf{g}(\mathbf{x}) \mid \mathbf{x} \in \mathbf{U}\} = \{(\mathbf{x}, \mathbf{y}) \mid \mathbf{x} \in \mathbf{U}, \mathbf{f}(\mathbf{x}, \mathbf{y}) = \mathbf{0}\}$$

which is continuously differentiable.

In this work, it is stated that there always exists a scalar, C^2 function $\mathrm{F} : \mathbf{R}^{\mathrm{n+m}} \to \mathbf{R}^1$ whose partial derivative with respect to the m parameters of $\mathbf{y}$ is $\mathbf{f} : \mathbf{R}^{n+m} \to \mathbf{R}^m$ and whose second partial derivative with respect to the m parameters of $\mathbf{y}$ is the Hessian according to the conditions of the theorem. Depending on the Hessian, the critical point where $\mathbf{f} = \mathbf{0}$ can be a local sub-minimum, a sub-maximum and may have several other local sub-geometries according to the Morse theorem. The implicit function collects the critical points of a local projection map.

Applications:

1. Any division $(\mathbf{x}, \mathbf{y}) = (\mathrm{x}_1, \ldots, \mathrm{x}_\mathrm{n}, \mathrm{y}_1, \ldots, \mathrm{y}_\mathrm{m})$ where the type of the critical point is a minimum for every value of $\mathbf{x}$ may lead to a hierarchical minimisation consisting of two kinds of smaller dimensional minimisations.

2. The graph of the implicit function assigns a transformed merit function (a section of the original merit function) which is minimised in the second minimisation with respect to $\mathbf{x}$ (during of which the $\mathbf{y}$-part is eliminated in every iteration step with respect to $\mathbf{x}$ by sub-minimisation with respect to $\mathbf{y}$).

3. The solution of hierarchical minimisation is identical to the solution of the original minimisation if the type of the critical point is a global minimum for every value of $\mathbf{x}$

The globally positive definite cases:

5. In case of strictly convex $\mathrm{F} : \mathbf{R}^{\mathrm{n+m}} \to \mathbf{R}^1$, for any division $(\mathbf{x}, \mathbf{y}) = (\mathrm{x}_1, \ldots, \mathrm{x}_\mathrm{n}, \mathrm{y}_1, \ldots, \mathrm{y}_\mathrm{m})$, the Hessian is globally positive definite for every value of $\mathbf{x}$.

6. In case of linearly dependent parameters, the second partial derivative with respect to the m parameters of $\mathbf{y}$ is globally positive definite for every value of $\mathbf{x}$.

No-dimension versions of classical theorems of combinatorial convexity in Banach spaces

Grigory Ivanov
Institute for Science and Technology Austria

Basic theorems in combinatorial and convex geometry such as Carathéodory's, Helly's and Tverberg's state different combinatorial properties of convex sets in $\mathbb{R}^d$. Their statements depend on the dimension d (these theorems can be used to characterize the dimension). The idea behind a "no-dimension" or an approximate version of the well-known theorems is to make them independent of the dimension. However, it comes at some cost – the approximation error. Such results are useful for different applications (for example, for computing Nash equilibria and for densest bipartite subgraph problem). Moreover, different problems on approximation of operators can be reformulated in the language of no-dimension theorems. We will discuss no-dimension versions of Carathéodory's, Tverberg's theorems together with their applications. Based on papers:

1. Ivanov, G. Approximate Carathéodory's Theorem in Uniformly Smooth Banach Spaces. Discrete Comput Geom **66**, 273–280 (2021). https://doi.org/10.1007/s00454-019-00130-w
2. Ivanov, G. (2021), No-dimension Tverberg's theorem and its corollaries in Banach spaces of type p. Bull. London Math. Soc., **53**, 631–641. https://doi.org/10.1112/blms.12449

Unitarily invariant valuations on convex functions

Jonas Knoerr
Technische Universität Wien (Austria)

I will present a characterization of all continuous, dually epi-translation invariant valuations on the space of finite-valued convex functions that are invariant under the unitary group. Every such valuation may be represented uniquely as a sum of functional versions of the hermitian intrinsic volumes. Moreover, these functionals admit an integral representation with respect to a family of certain measure-valued valuations that includes the real and complex Monge-Ampère operator.

Projecting points between convex sets

Eva Kopecká
University of Innsbruck (Austria)

Let X and Y be two closed subspaces of a Hilbert space. If we send a point back and forth between them by orthogonal projection, the iterates converge to the projection of the point onto the intersection of X and Y. If, more generally, X and Y are just two intersecting closed and convex sets and we send the point back and forth between them by the nearest point projection, the sequence so obtained does not have to converge any more.

Given a finite family of closed convex sets with non-empty intersection, we again obtain a sequence by iterating the nearest point projection of a point onto these sets. We will relate the convergence of this sequence to the geometry of the convex sets and to the choice of the order in which we project onto them. For example, the symmetry of the sets, and projecting in every step on the remotest convex set implies convergence.

Inner and outer Mean Radii of Convex Bodies

Nico Lombardi
Technische Universität Wien (Austria)
(Joint work with Eugenia Saorín Gómez)

We are going to introduce the notions of inner and outer mean radii of convex bodies, with respect to projections and sections, that are generalizations of the notions of inradius and circumradius of convex bodies. They belong to a bigger family of radii for convex bodies and we are going to see the similarities and differences between them. After showing some basic properties of the radii, we will consider the behavior of the radii with respect to geometric symmetrizations of convex bodies, as Steiner and Minkowski symmetrizations, and with respect to the intrinsic volumes.

On discrete log-Brunn-Minkowski type inequalities

Eduardo Lucas
Universidad de Murcia (Spain)
(Joint work with María A. Hernández Cifre)

The conjectured log-Brunn-Minkowski inequality says that the volume of centrally symmetric convex bodies $K, L \subset \mathbb{R}^n$ satisfies $\mathrm{vol}\big((1-\lambda)\cdot K +_0 \lambda \cdot L\big) \geq \mathrm{vol}(K)^{1-\lambda}\mathrm{vol}(L)^{\lambda}$, $\lambda \in (0,1)$, and is known to be true in the plane and for particular classes of n-dimensional symmetric convex bodies. In this talk we discuss some new discrete log-Brunn-Minkowski type inequalities for the lattice point enumerator $\mathrm{G_n}(\cdot) = |\cdot \cap \mathbb{Z}^n|$. Among others, we show that if $K, L \subset \mathbb{R}^n$ are unconditional convex bodies and $\lambda \in (0,1)$, then

$$\mathrm{G_n}\biggl((1-\lambda)\cdot\bigl(K+C_n\bigr) +_0 \lambda\cdot\bigl(L+C_n\bigr)+\Bigl(-\frac{1}{2},\frac{1}{2}\Bigr)^n\biggr) \geq \mathrm{G_n}(K)^{1-\lambda}\mathrm{G_n}(L)^{\lambda},$$

where $C_n = [-1/2, 1/2]^n$. Neither C_n nor $(-1/2, 1/2)^n$ can be removed. Furthermore, it implies the (volume) log-Brunn-Minkowski inequality for unconditional convex bodies. The corresponding results in the L_p setting for $0 < p < 1$ are also discussed.

On extensions of Grünbaum's inequality

Francisco Marín Sola
Universidad de Murcia (Spain)
(Partly joint work with Jesús Yepes Nicolás)

Given a compact set $K \subset \mathbb{R}^n$ of positive volume, if K is convex with centroid at the origin, then, a classical result by Grünbaum says that one can find a lower bound for the ratio $\mathrm{vol}(K^-)/\mathrm{vol}(K)$ depending only on the dimension of K, where K^- denotes the intersection of K with a halfspace bounded by a hyperplane passing through its centroid.

In this talk, among other results, we show that fixing the hyperplane H, one can find a sharp lower bound for the ratio $\mathrm{vol}(K^-)/\mathrm{vol}(K)$ depending on the concavity nature of the function that gives the volumes of cross-sections (parallel to H) of K. When K is convex, this inequality recovers the previous result by Grünbaum. To this respect, we also show that the log-concave case is the limit concavity assumption for such a generalization of Grünbaum's inequality.

Finally, we will give an alternative proof for the functional version of Grünbaum's inequality showing also how both the functional and geometric results are equivalent.

Hyperbolic crystallography by fundamental polyhedra and extremum problems

To the Memory of János Bolyai on 220th Anniversary of His Birth

Emil Molnár
Budapest University of Technology and Economics (Hungary)
(Joint work with István Prok and Jenő Szirmai)

A compact manifold of constant curvature is called space form. This concept can naturally be extended to any space X of the 8 Thurston's (simply connected homogeneous Riemann) 3-geometry: Thus, we look for a fixed-point-free isometry group $\mathbf{G}$, acting on X with compact fundamental domain $\mathcal{F} = X/\mathbf{G}$, endowed by appropriate face pairing identifications.

It turned out that the previous (1984-88) initiative of the first author, constructing new hyperbolic space forms (football manifolds), with István PROK's computer implementations [13, 14] and Jenő SZIRMAI's numerical computations (e.g. in [8], have got applications in crystallography, e.g. as fullerenes. Nowadays we (István Prok, Jenő Szirmai, Andrei Vesnin, Alberto Cavicchioli, Fulvia Spaggiari) found further space forms (also in other Thurston spaces, not only in $\mathbf{E}^3$, $\mathbf{S}^3$ and $\mathbf{H}^3$, but also in $\mathbf{S}^2\times\mathbf{R}$, $\mathbf{H}^2\times\mathbf{R}$, $\mathbf{Nil}$, $\widetilde{\mathbf{SL}_2\mathbf{R}}$, $\mathbf{Sol}$). Furthermore, extremum problems and other possible applications, as infinite series $Cw(2z, 1 < z \text{ odd})$ of nanotubes with z-rotational symmetry are foreseen.

Maybe, our experience space in small size can be non-Euclidean as well!!!

Further new $\mathbf{H}^3$ results are related to surgeries of the famous *Gieseking ideal simplex manifold* by using computer figures in C_∞. The other 3 double-simplex manifolds e.g. in [2], [16] found also by I. Prok with his computer program [13, 14], can be described in Poincaré half-space model of $\mathbf{H}^3$ and by the complex projective line C_∞. The Dehn type surgery leads to compact manifolds and cone manifolds (especially orbifolds) through complex 2×2 matrix algebra and attractive pictures. Meanwhile [10, 11] we found interesting phenomena, related with singular geodesics of rotational order $k = 2, 3, \ldots$ and simplex cone manifolds $S(k)$ whose volume tending to zero if k goes to infinity. These are related to theorem of D. A. Kazhdan and G. A. Margulis [3] and with the work of W. P. Thurston [15], describing the geometric convergence of orbifolds under large Dehn fillings.

We completed our discussion and derived the above cone manifold series (Gies.1 and Gies.2) by computer figures as well – in two geometrically equivalent form, by the half turn symmetry of any ideal simplex. Moreover, we obtained a second orbifold series (Gies.3 and 4), tending to the regular ideal simplex as the original Gieseking manifold. We have already extended this method onto the 3 ideal doublesimplex manifolds in our newer publication

[11], being indicated in this presentation as well. Our computer figures on C_∞ (the complex number plane) play important roles in exploring the exact situations. We can see more related results in [1], [4], [5], [6], [7], [8], [9], [10], [11], [12], [16].

References

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Functional Intrinsic Volumes

Fabian Mussnig
Technische Universität Wien (Austria)
(Joint work with Andrea Colesanti and Monika Ludwig)

We consider functional intrinsic volumes on super-coercive convex functions. For a convex, lower semicontinuous function $u : \mathbb{R}^n \to (-\infty, \infty]$ such that $\lim_{|x|\to\infty} \frac{u(x)}{|x|} = +\infty$ these operators are of the form

$$u \mapsto \int_{\mathbb{R}^n} \zeta(|\nabla u(x)|)[\mathrm{D}^2 u(x)]_{n-i} \,\mathrm{d}x$$

if in addition $u \in C^2_+(\mathbb{R}^n)$. Here, $i \in \{0, \dots, n\}$ and we denote by $[\mathrm{D}^2 u(x)]_{n-i}$ the $(n-i)$th elementary symmetric function of the eigenvalues of the Hessian matrix $\mathrm{D}^2 u(x)$ at $x \in \mathbb{R}^n$. Furthermore, $\zeta : (0, \infty) \to \mathbb{R}$ is continuous with bounded support and needs to satisfy additional regularity conditions which depend on i and n.

We will discuss similarities with the classical intrinsic volumes by looking at characterization results, equivalent representations and integral geometry. Furthermore, we will see that some classical results can be retrieved from the new ones.

Best and random approximations with generalized disc–polygons

Kinga Nagy
University of Szeged (Hungary)
(Joint work with Viktor Vígh)

In this contribution, we consider the asymptotic behaviour of the distance between a convex disc K with sufficiently smooth boundary, and its approximating n-gons, as the number of vertices tends to infinity. We consider two constructions: the best approximating inscribed n-gon of K is the one with maximal area; and a random inscribed n-gon of K is the convex hull of n i.i.d. random points chosen from the boundary of K. The asymptotic behaviour of the area deviation of K and the n-gon depend in both cases on the same, geometric limit. The best and random approximating n-gons can be similarly defined in the circumscribed case.

We generalize the existing results on linear and spindle convexity to the so-called L-convexity. In the case of inscribed L-polygons, we prove similar asymptotic formulae by generalizing the geometric limits. Then we introduce an L-convex duality, consider its properties, and use them to prove the formulae for the circumscribed cases.

Löwner's problem for log-concave functions

Márton Naszódi
Loránd Eötvös University, Budapest (Hungary)
(Joint work with Grigory Ivanov and Igor Tsiutsiurupa)

The class of logarithmically concave functions is a natural extension of the class of convex sets in Euclidean d-space. Several notions and results on convex sets have been extended to this wider class. We study how the problem of the smallest volume affine image of a given convex body L that contains another given convex body K can be phrased and solved for functions.

Equidistant sets and convex bodies

Márk Oláh
University of Debrecen (Hungary)
(Joint work with Csaba Vincze)

The equidistant set of two nonempty subsets K and L (called focal sets) in the Euclidean space $\mathbb{R}^n$ is the set of all points that have the same distance from both K and L, measured with the infimum metric. The most well-known examples of equidistant sets in the plane (apart from segment and angle bisectors) are the classical conics, which means that we can consider equidistant sets as a kind of their generalizations.

But the class of equidistant sets is much wider than this. In [1], Vincze proved that all closed convex planar curves in the plane are equidistant sets, by describing a method to generate the two focal sets for any closed convex polygon, and approximating general closed convex planar curves by these polygons and using the continuity theorem of Ponce and Santibanez (see [2]). It turns out that this result is not bound to dimension two: we prove that in any dimensions, the boundary of any convex polytope can be generated as an equidistant set, and by applying the approximation technique from above, the same is true for any convex body (see [3]). Thus, in some sense, equidistancy is a generalization of the notion of convexity.

By this method, we can generate any convex polytope as the equidistant set of an inner singleton and an outer focal set having as many elements as the number of the facets. Introducing the notion of equidistant polytopes, classified by the cardinalities of their focal sets, we can say that all convex polytopes having m facets are equidistant polytopes of type $(1, m)$. Stepping out of the discrete case, the authors of [2] posed the question to characterize all closed sets that can be realized as the equidistant set of two *connected* disjoint closed sets. We show that by connecting the points in the outer focal set in an adequate way, we can achieve such a focal set for any convex polytope.

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Random approximations by generalized disc-polygons

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For two convex discs K and L, we say that K is L-convex if it is equal to the intersection of all translates of L that contain K. We study the following probability model: let K and L be C^2_+ smooth convex discs such that K is L-convex. Select n i.i.d. uniform random points $x_1, \ldots, x_n$ from K, and consider the intersection $K_{(n)}$ of all translates of L that contain all of $x_1, \ldots, x_n$. The set $K_{(n)}$ is a random L-convex polygon in K. We study the expectation of the number of vertices and missed area of $K_{(n)}$ as n tends to infinity. We consider two special cases: in the first case we assume that the curvatures of K and L can be bounded away from each other uniformly, in the other case we let $K = L$.

Large deviation principles for lacunary trigonometric sums

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Classical results of Kac, Erdős and Gal, and Salem and Zygmund show that lacunary trigonometric sums behave in many ways like sums of independent random variables. We present large deviation principles in this setting, showing that quite surprisingly and contrary to the classical results, the behavior is very sensitive to the arithmetic properties of the Hadamard gap sequence.

The isodiametric problem in spaces of constant curvature and its stability

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The isodiametric inequality in the Euclidean space was proved by Bieberbach and Urysohn; namely, balls maximize the volume of a convex body of given diameter. We verify the analogous statement in the spherical and hyperbolic spaces. In addition, we prove a stability version of this statement in each of the three types of spaces of constant curvature.

Remarks on certain parallel linear behavior of mixed volumes and mixed discriminants

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(Joint work with C. de Vries and N. Lombardi)

Let $\mathcal{M}^n$ denote the set of real symmetric positive semidefinite $n\times n$ matrices. The mixed discriminant $D:(\mathcal{M}^n)^n\longrightarrow\mathbb{R}$ is the unique symmetric function for which

$$\det(\lambda_1 A_1+\ldots\lambda_m A_m)=\sum_{i_1,\ldots,i_n=1}^{m}\lambda_{i_1}\ldots\lambda_{i_n}\,D(A_{i_1},\ldots,A_{i_n})\qquad(1)$$

for $m\in\mathbb{N}$, $\lambda_1,\ldots,\lambda_m\geq 0$, and $A_1,\ldots,A_m\in\mathcal{M}^n$.

Panov proved that the following two statements are equivalent for positive semidefinite symmetric matrices $A_1,\ldots,A_m\in\mathcal{M}^n$. For a matrix $A\in\mathcal{M}^n$, let $P(A)$ denote the linear subspace of $\mathbb{R}^n$ spanned by the eigenvectors of A having positive associated eigenvalues.

i) $D(A_1, \ldots, A_n) > 0$,

ii) If $K_i = \dim P(A_i) \cap B^n$, where B^n is the unit ball in $\mathbb{R}^n$, then

$$V(K_1, \ldots, K_n) > 0.$$

Here, as usual, $V(K_1, \ldots, K_n)$ denotes the mixed volume of the convex bodies $K_1, \ldots, K_n$.

Florentin, Milman and Schneider provided a characterization of mixed discriminants among all functions $(\mathcal{M}^n)^n \longrightarrow \mathbb{R}$ by means of the non-negativity, additivity in each variable and the fact that the function is zero if any two arguments are proportional matrices of rank one.

In the setting of convex bodies, Milman and Schneider characterized the mixed volume of centrally symmetric convex bodies in $\mathbb{R}^n$ was as the only function, up to a multiplicative constant, of n centrally symmetric convex bodies which is Minkowski additive and increasing (with respect to set inclusion) in each variable and which vanishes if two of its arguments are parallel segments.

The latter shows a strong analogy of defining, structural and behavioral aspects of mixed discriminants and mixed volumes. There are some differences though, which seem not to allow to change the convex bodies framework to the symmetric positive semidefinite matrices one.

In this talk we will thoroughly introduce mixed discriminants, mixed volumes and the known connections among those. Then, we will consider some refinement of inequalities, such as the Aleksandrov-Fenchel inequality, which are known in the geometrical setting, aiming for an analogous version within the context of mixed discriminants, in a broad sense. We will address quantities associated to positive semidefinite matrices, resembling those naturally appearing in the geometric setting.

Totally separable translative packings of a convex body

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(Joint work with Márton Naszódi)

A packing of translates of a convex body is called *separable* if any two translates can be separated by a hyperplane that does not intersect the interior of any translate of the packing. This notion was introduced by Gábor Fejes Tóth and László Fejes Tóth in 1973, and studied mostly by considering the density of such packings. More recently, Károly Bezdek and others considered the combinatorial properties of the contact graphs of totally separable packings. In a *contact graph* of a packing, we consider the translates to be the vertices, and join two vertices when the two translates touch. Some interesting combinatorial properties of contact graphs are their maximum degree (Hadwiger number), minimum degree, and total number of edges (contact number). We will discuss recent results about these numbers for separable packings in the plane and in higher dimensions.

Mass-distribution of partitions of convex bodies by hyperplanes

Jesús Yepes Nicolás
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(Joint work with David Alonso-Gutiérrez, Francisco Marín Sola and Javier Martín Goñi)

A classical result by Grünbaum provides the extremal mass ratio for the portions obtained when cutting a convex body by a hyperplane passing through its centroid.

In this work we will discuss various generalizations of this result to the case of cuts (by hyperplanes) through other particular points.

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