Functional Intrinsic Volumes

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We consider functional intrinsic volumes on super-coercive convex functions. For a convex, lower semicontinuous function $u : \mathbb{R}^n \to (-\infty, \infty]$ such that $\lim_{|x|\to\infty} \frac{u(x)}{|x|} = +\infty$ these operators are of the form

$$u \mapsto \int_{\mathbb{R}^n} \zeta(|\nabla u(x)|)[\mathrm{D}^2 u(x)]_{n-i}\,\mathrm{d}x$$

if in addition $u \in C^2_+(\mathbb{R}^n)$. Here, $i \in \{0, \dots, n\}$ and we denote by $[\mathrm{D}^2 u(x)]_{n-i}$ the $(n-i)$th elementary symmetric function of the eigenvalues of the Hessian matrix $\mathrm{D}^2 u(x)$ at $x \in \mathbb{R}^n$. Furthermore, $\zeta : (0, \infty) \to \mathbb{R}$ is continuous with bounded support and needs to satisfy additional regularity conditions which depend on i and n.

We will discuss similarities with the classical intrinsic volumes by looking at characterization results, equivalent representations and integral geometry. Furthermore, we will see that some classical results can be retrieved from the new ones.