Contents

2-affine complete algebras need not be affine complete

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Given a finite algebra $\mathbf{A}$, we study the clone $\mathrm{Comp}(\mathbf{A})$ of all congruence preserving functions on $\mathbf{A}$, and the clone $\mathrm{Pol}(\mathbf{A})$ of all polynomial functions on $\mathbf{A}$. We call an algebra n-affine complete iff every n-ary congruence preserving function is polynomial.

For each $k \in \mathbb{N}$, we exhibit an algebra that is k-affine complete, but not $(k+1)$-affine complete.

However, as a consequence of a Theorem by J. Hagemann and Chr. Herrmann, we know that if each homomorphic image of a finite algebra $\mathbf{A}$ in a congruence permutable variety is 2-affine complete, then $\mathbf{A}$ is k-affine complete for all $k \in \mathbb{N}$.

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Tensor products of compact rings

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We introduce the notion of a tensor product of compact Φ-modules over a discrete commutative ring Φ with identity. Using this notion we prove the existence of tensor products in the category of compact zero-dimensional rings.

Definition. Let Φ be a commutative discrete ring with identity and $A_1, ..., A_n$ compact Φ-modules. A pair (C, π), where C is a compact Φ-module and π is a n-linear mapping of $A_1 \times ... \times A_n$ in C is called a tensor product of $A_1, ..., A_n$ provided for every n-linear mapping $f : A_1 \times ... \times A_n \to D$ in a compact Φ-module D there exists a unique continuous Φ-homomorphism $\widehat{f} : C \to D$ such that $\alpha = \widehat{f} \circ \pi$.

Theorem 1. *If $A_1, ..., A_n$ are compact unitary Φ-modules then there exists the tensor product $A_1 \otimes ... \otimes A_n$.*

Theorem 2. *If Φ is a commutative discrete ring with identity, A, B, C some unitary compact zero-dimensional Φ-modules, then there exists a unique topological Φ-isomorphism $(A \otimes B) \otimes C \to A \otimes B \otimes C$ for which $(a \otimes b) \otimes c \mapsto a \otimes b \otimes c$.*

Theorem 3. *If A and B are two zero-dimensional Φ-algebras over a discrete commutative ring Φ with identity then there exists a structure of compact Φ-algebra on $A \otimes_\Phi B$ such that $(a \otimes b)(a' \otimes b') = aa' \otimes bb'$ for $a, a' \in A$, $b, b' \in B$.*

Joint work with Ursul Mihail (Department of Mathematics, University of Oradea, Romania).

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Bela Lettres

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I will discuss some of the mathematics contained in the correspondence that I have had with Bela Csakany over the past 25 years. The context of these letters will be reviewed and the development of some specific themes and threads contained in them will be traced.

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The lattice of projection operators of a subtractive nearsemilattice

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A nearsemilattice is a poset in which every principal order ideal is a join semilattice. A nearsemilattice A is subtractive if it has the least element 0 and is equipped with a total binary operation $-$ satisfying the following axioms:

(i) if $y \vee z$ exists, then $x - y \leq z$ iff $x \leq y \vee z$,

(ii) if $x - y \leq z$, then $x - z \leq y$.

A projection operator on A is an idempotent endomorphism f such that $f(x) \leq x$. We characterize the kernel ideals $f^{-1}(0)$ of projection operators and prove that these operators form, under the pointwise ordering, a bounded lattice which is dually embeddable in the (distributive) lattice of ideals of A.

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On the Berman Conjecture for Finite Semigroups

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If A is any (finite) algebra, by $p_n(A)$ we denote the number of all n-ary term operations which depend on all of its variables ($p_0(A)$ denotes the number of all constant unary term operations of A). The principal goal of the theory of p_n-sequences is to characterize those sequences of non-negative integers which are representable as the p_n-sequence of some algebra (or, possibly, of an algebra of some specific kind, e.g. of a semigroup). Hence, investigations aiming to establish the way in which the numerical properties of sequences influence the structure of corresponding algebras, have a central importance in this theory.

Towards this distant goal, in 1986 J. Berman formulated an interesting conjecture, claiming that the p_n-sequence of any finite algebra is either bounded above by a constant, or eventually strictly increasing. Ten years later, this conjecture was shown by R. Willard to be false. Still, the Berman Conjecture (BC) holds for a vast number of 'natural' algebras, such as monoids, groups, rings, modules, lattices, Boolean algebras, etc. Our interest here is in the restricted BC for semigroups. We present several wide classes of finite semigroups satisfying the property indicated by the BC. For example, these classes include all globally idempotent ($S^2 = S$) and all commutative finite semigroups.

Joint work with Igor Dolinka (Institute of Mathematics and Informatics, University of Novi Sad), Nikola Ruškuc (Mathematical Institute, University of St Andrews).

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Optimal Mal'tsev conditions for modular congruence lattice identities

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It was proved in [1] that in every congruence modular variety we have $\alpha \cap \beta^* \subseteq (\alpha \cap \beta)^*$ for any tolerances α and β. Here * stands for transitive closure. Based on [1], S. Radeleczki and K. Kearnes, independently, derived an even more useful property of tolerances in congruence modular varieties: $\alpha^* \cap \beta^* = (\alpha \cap \beta)^*$. This property will be called *tolerance intersection property*, TIP in short.

Based on TIP, it was proved in [2] that for an arbitrary lattice identity implying modularity (or at least congruence modularity) there exists a Mal'tsev condition such that the identity holds in congruence lattices of algebras of a variety if and only if the variety satisfies the corresponding Mal'tsev condition. However, the Mal'tsev condition constructed in [2] is not the simplest known one in general. In [3] we improve this result by constructing the best Mal'tsev condition and various related conditions. In particular, if p and q are lattice terms and V is a congruence modular variety then the lattice identity $p \leq q$ holds for congruences of V iff $p_2 \subseteq q$ holds for congruences of V where p_2 comes from p by substituting $x \circ y$ for $x \vee y$ everywhere.

As an application, Lipparini [3] gave a particularly easy new proof of Freese and Jónsson's result stating that modular congruence varieties are Arguesian, and he strengthened this result by replacing "Arguesian" by "higher Arguesian" in M. Haiman's sense.

Using TIP and commutator theory, Lipparini [3] showed that if p is an n-ary lattice term and $\alpha_1, \ldots, \alpha_n$ are congruences in an arbitrary congruence modular variety then

$$p(\alpha_1, \ldots, \alpha_n) = p^{(d)}(\alpha_1, \ldots, \alpha_n) \circ p_2(\alpha_1, \ldots, \alpha_n)$$

where $p^{(d)}$ is the disjunctive normal form of p, computed as if we were in a distributive lattice. This result reminds us Gumm's famous paper "Congruence modularity is permutability composed with distributivity".

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[3] G. Czédli, E. K. Horváth and P. Lipparini: Optimal Mal'tsev conditions in modular congruence varieties, *Algebra Universalis*, submitted.

Joint work with Eszter K. Horváth and Paolo Lipparini.

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The geometry monoid of an identity

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For each algebraic identity I (or family of algebraic identities), natural questions are the word problem of I, i.e., the description of an algorithm recognizing whether two terms are forced to be equal by I, and the construction of concrete realizations for the free systems in the equational variety defined by I—and, more generally, of concrete examples of systems satisfying I. Of course, answering such questions depends on the considered identity in an essential way, and it seems hopeless to find a uniform method that works for all identities.

Due to its connection with iterations of elementary embeddings in set theory and with braid groups in low dimensional topology, the left self-distributivity identity $x(yz) = (xy)(xz)$ has recently received some attention, and the above questions have been solved by introducing a specific monoid that captures some geometrical properties—and turns out to be connected with Artin's braid groups.

A similar geometry monoid can be defined in the case of associativity; then, the monoid is essentially R. Thompson's group F, and it is connected with the well known Mac Lane–Stasheff pentagon.

The geometry monoid exists for every family of identities, but, in general, it is a complicated object, and, as most of the technical properties used in the cases of associativity and left self-distributivity rely on specific properties of these identities, one could doubt that the method applies to other identities.

Actually, it does. Here, we shall consider the case of Identity (CD): $x(yz) = (xy)(yz)$. This identity has probably little interest in itself, but it should be clear that the subject of the talk is not really that particular identity, but rather the method we use for studying it, namely investigating the corresponding geometry monoid.

The results we obtain are:
– a primitive recursive solution for the word problem of Identity (CD);
– a realization of the free monogenic CD-system in a quotient of the geometry monoid.

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Endomorphism Monoid of the Random Graph

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Erdös and Rényi proved in 1963 a rather surprising result that there is a unique countably infinite random graph. Over the years, there has been much insight into the structure of $Aut(\mathcal{R})$, its automorphism group.

In this talk, we will present some recent results on the properties of the endomorphism monoid of $\mathcal{R}$, obtained in the joint with work with A. Bonato. In particular, we shall discuss the Rees ordering of $End(\mathcal{R})$ which turns out to embed the ordering of the rationals and contains uncountably many minimal elements.

Joint work with Anthony Bonato (Wilfrid Laurier University).

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Congruence regularity and its generalizations

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In regular algebras (see [8]), every congruence is uniquely determined by each of its classes, thus also every class containing a given element c is uniquely determined by a given class. For weakly regular algebras with a constant 0, every class is determined by the class containing 0 (see e.g. [9], [10]) and, vice versa, for locally regular algebras (introduced by the first author in [1]), the class containing 0 is determined by every congruence class. This scheme motivated us to find a general concept of dependency of congruence classes which can be applied not only to the afore mentioned cases but also for other modifications of congruence regularity (see [2], [3], [4], [5], [6], [7]).

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On equational classes of Boolean functions

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Certain classes of Boolean functions, including all Post classes, can be characterized by functional equations. Recent work by this group of authors and by Pippenger has addressed issues such as necessary and sufficient conditions for such characterizations to exist, connections with preservation of relations, connection with the HSP Theorem, minimum number of equations / variables required, and constraints on the form of the equations allowed. Published and current results related to these issues will be discussed.

Joint work with Miguel Couceiro (Tampere U of Technology and U of Tampere, Finland), Oya Ekin (Bilkent, Ankara), Peter L. Hammer (Rutgers, New Jersey), Lisa Hellerstein (Polytechnic University, New York) and Grant Pogosyan (International Christian University, Tokyo).

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Independence notions from a general algebraic point of view

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In 1958 E. Marczewski introduced a general notion of independence, which contained as special cases majority of independence notions used in various branches of mathematics. A non-empty set I of the carrier A of an algebra is called M-independent if equality of two term operations f and g of the considered algebra on any finite system of different elements of I implies $f = g$ in A. There are several interesting results on this notion of independence. However the important scheme of M-independence is not enough wide to cover the stochastic independence, the independence in projective spaces and some others. This is why some notions weaker than the M-independence were developed. The notion of independence with respect to family Q of mappings (defined on subsets of A) into A, Q-independence for short, is a common way of defining almost all known notions of independences. There exists an interesting Galois correspondence between families Q of mappings and families of Q-independent sets. In our talk after a brief survey of these topics we will mainly concentrate on a few easily formulated and interesting results.

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Standard topological quasi-varieties – examples and problems

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The introduction of standard topological quasi-varieties has been motivated by the theory of natural dualities, which provides a tight connection between a given quasi-variety of algebras and its dual topological quasi-variety. We initiate a program of study to determine which choices of the topological quasi-variety are standard and which are not. We say that a topological quasi-variety is standard if, in an appropriate sense, there is a nice axiomatic description of its members which allows us to recognize them by looking only at their finite substructures. The study lies at the confluence of algebra, topology and mathematical logic, and has been initiated by the Duality Research Group at La Trobe University (Melbourne) in 2000.

Joint work with David M. Clark (SUNY, New Paltz), Brian A. Davey (La Trobe University, Melbourne), Jane G. Pitkethly (La Trobe University, Melbourne), Rashed M. Talukder (La Trobe University, Melbourne).

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Representation Theory of Cylindric Lattices

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Andréka, Comer, Németi and van Benthem started to investigate algebraic counterparts of first-order logic without negation (see eg. [1]).

They introduced different classes of distributive cylindric lattices (RCA^{-}_{α}, $RCA^{\pm}_{\alpha}$) as algebraic counterparts of these logics.

We answer an open problem of Comer (problem 12 in [1]) about axiomatizability properties of RDf^{-}_{α} by developing a new representation theory for these classes. We also investigate the axiomatizability properties of the generated varieties.

Here we will find the interesting and unexcepted example of a class of two-dimensonal cylindric reducts having highly complex axiomatizability properties ($\mathbf{H}RDf^{\pm}_{2}$).

The material of the talk is partly covered by [2].

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Tolerance intersection property in congruence modular varieties

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A proof will be presented to the fact that in congruence modular varieties for each tolerances of an algebra

$$\Gamma^* \cap \Phi^* = (\Gamma \cap \Phi)^*$$

holds, where Γ^* means the transitive closure of the tolerance relation Γ. The proof is based on the characterication of congruence modularity by A. Day.

Joint work with Gábor Czédli.

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Membership problems for finite algebras

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While the class of finite entropic groupoids (homomorphic images of medial cancellation groupoids) is recursive, the same is not true for the class of finite partial entropic groupoids.

Joint work with Miklós Maróti.

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Characterizations of Standard Element in Posets

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The concepts of distributive and semidistributive pairs in posets are introduced and some properties of these pairs and their relationships with modular pairs in posets are discussed. A Fundamental Characterization Theorem for standard elements and several characterizations in SSC, SSC^* and atomistic posets are established.

Joint work with B. N. Waphare (Department of Mathematics, University of Pune, Pune.).

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A generalization of closures

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A generalization of the closures is defined for an arbitrary poset. Its property is studied, interesting examples are given and some extremal problems are solved for this concept.

Joint work with János Demetrovics (Computer and Automatization Institute, Hungar. Acad. Sci.), Attila Sali (Rényi Institute, Hungar. Acad. Sci.).

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Hausdorff properties of topological algebras

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We discuss the problem of characterizing the varieties whose T_0 topological algebras are Hausdorff.

Joint work with Luis Sequeira.

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Reducibility Number

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The notion of reducibility number is introduced and studied. The reducibility numbers for power set $\mathbf{2}^n$, of an n-set ($n \geq 2$) with respect to the classes of distributive lattices, modular lattices and Boolean lattices are calculated. Also, new light is thrown on the reducibility considerations as well as on the relationships between some groups G and their subgroup lattices $\mathcal{L}(G)$. The class of pseudocomplemented u-posets is shown to be reducible. Deletable elements in semidistributive posets are characterised.

Joint work with B. N. Waphare (Dept. of Mathematics, University of Pune), N. K. Thakare (Dept. of Mathematics, University of Pune).

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Nilpotence, twin groups, and an application

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Among the various extensions of tame congruence theory, one of the most important ones is the theory of nilpotence. Strongly related to this is the concept of the twin group, consisting of the twins of the identity map on a subset of the algebra. These groups help to describe various aspects of nilpotence, and are related to such important properties as residual smallness, or free spectra. The talk will give a short introduction to these concepts, which will hopefully be useful to newcomers to this area, too. Then, as an application, we shall proceed to see how these concepts help to investigate algebras generating a variety with definable principal congruences.

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Regular solid varieties of semigroups

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In order to get a better insight into the lattice of all varieties of semigroups we consider complete lattices, namely the lattices of all M-solid varieties of semigroups, where M is a monoid of hypersubstitutions. A hypersubstitution is a mapping which maps the binary operation symbol to a binary term. Special hypersubstitutions are regular hypersubstitutions. They map the binary operation symbol to binary terms which are not unary. The set of all regular hypersubstitutions forms a monoid Reg. We want to describe the lattice of all Reg-solid varieties of semigroups, in particular, we characterize the greatest one and the commutative part.

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Linear functional equations on almost quasigroups: totally debalanced equations

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A quasigroup may be characterized by the property that the (left or right) translation by any element is permutation. If we allow that some translations may be constant functions, we get so called *almost quasigroups*. The quasigroups are characterized as isotopes of loops. Similarly we can characterize almost quasigroups as isotopes of either loops or loops with external zero added.

A special class of linear functional equations on almost quasigroups is considered. They are called *totally debalanced* and characterized by a single appearance of variables (both object and functional) on the left side of the given equation $u = b$, while the right hand side (b) is a constant.

The general solution of such functional equation is described in terms of the structure of the tree of the term u.

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Equiconvergence in the realm of suitable posets

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Equiconvergence is being considered as a special kind of filterunitopies, i.e. we axiomatize uniform filters converging to suitable points. Thus, simple convergence as well as uniform convergence bothly can be described in such a manner. Moreover, it turns out that the corresponding category is set-like enough to allow constructions of special objects such as function spaces and universal one-point extensions. Hence, our new constructed category satisfy the well known set of exponential laws, and additionally we have that final epi-sinks are hereditary.

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On some relation between clones and monoids

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For a set S of multi-variable operations on a k-element set E_k, the centralizer S^* is the set of operations which commute with all operations in S. In this talk we concentrate on transformation monoids of unary operations on E_k and consider centralizers of such monoids. In particular, our interest is in finding monoids whose centralizer is the trivial (least) clone J_k.

First we define an ascending sequence of monoids which contain the symmetric group S_k on E_k and show that the centralizers of most of the monoids in this sequence coincides with J_k. Then we give examples of smaller monoids whose centralizers are J_k as well. The former result is more natural and the latter result is less natural when we consider the relation induced by $*$ forms a Galois connection between monoids and clones. In the course of discussion the Kuznetsov criterion is effectively used.

Joint work with Ivo G. Rosenberg.

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On Power Structures

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In general, the *power structure* of a structure $\mathcal{A}$ with the universe A is an appropriate structure defined on the power set $\mathcal{P}(A)$ such that the structure on elements from A are "lifted", in some way, to the subsets of A. We investigate various specific problems which arise from the general question in this topic: How the properties of the structure $\mathcal{A}$ have been changed when we lifted $\mathcal{A}$ to its power structure?

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The Whitney numbers and the product of partially ordered sets

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Let consider the Cartesian product of finite partially ordered sets. Our aim is to estimate the width of it (the maximum over numbers of elements of all antichains in this partially ordered set).

It is appear, that in the case the product of chains and some subsets of it, the width is realized by one of the Whitney numbers - the numbers of elements of the same rang. We calculate Whitney numbers of the product of three chains and some subset of it and product of four chains and as a consequence we get the width of these partially ordered sets.

Joint work with Aleksander Rutkowski.

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Operators on classes of coalgebras

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A class K of T-coalgebras is called a covariety if $K = SH\Sigma(K)$. We show that starting from the coalgebraic class operators S, H and Σ one can form exactly 12 different class operators. The characterisation holds for an arbitrary SET-endofunctor T. We also describe the class operators for SET-endofunctors with some special properties (e.g. those that preserve weak pullbacks).

Joint work with Boza Tasic (University of Waterloo, Canada).

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Equational Bounds:
Determining Finite Algebras in Finitely Generated Varieties

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With every variety $\mathcal{V}$ we associate a function $\beta_{\mathcal{V}}$ on the positive integers so that $\beta_{\mathcal{V}}(n)$ is the least positive integer k such that for every algebra $\mathbf{B}$ with exactly n elements

$$\mathbf{B} \in \mathcal{V}$$

if and only if

every equation of length no more than k which is true in $\mathcal{V}$ is true in $\mathbf{B}$.

The function $\beta_{\mathcal{V}}$ is called the **equational bound of** $\mathcal{V}$. By the equational bound $\beta_{\mathbf{A}}$ of the algebra $\mathbf{A}$ we mean the equational bound of the variety generated by $\mathbf{A}$.

Here we investigate the assymptotic behavior of equational bounds for finite algebras. It is clear that if $\mathbf{A}$ is finitely based, then $\beta_{\mathbf{A}}$ is eventually dominated by a constant function. So our investigation must focus on nonfinitely based algebras. We show in many cases that the equational bound is eventually dominated by a linear function. On the other hand, we describe a general method for constructing finite algebras with equational bounds that eventually dominate every sublinear function.

Joint work with Zoltán Székely (Gallaudet University).

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Finite lattices as lattices of R-congruences of finite unars and Abelian groups

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Let R be quasivariety of algebras. A congruence θ on algebra A is called R-congruence if $A\backslash\theta$ belongs to R. A set $Con_R A$ of all R-congruences of algebra A is algebraic lattice and is called a lattice of R-congruences on algebra A. If R is variety then this lattice is usual lattice of congruences $ConA$. It's well known problem [1]: Is any finite lattice is isomorphed to a lattice of congruence of finite algebra? The problem is still open. There exist a lot of results concerning this problem. We proof that the problem have positive solution for lattices of $R-$congruences.

THEOREM 1. *Let L be finite lattice. Then there exist locally finite quasivariety R (K) of unars (groups) and finite unar U (group G) such that L is isomorphed to* $\mathrm{Con}_R U$ $(\mathrm{Con}_K A)$.

Subclass K of quasivariety R of algebras is called R-variety if $K = R \cap V$ for some variety V. A set $L_V(R)$ of all R-varieties of quasivariety R is co-algebraic lattice and is called R-varieties lattice.If R is variety then this lattice is usual varieties lattice. In [2] was prooved that there exist locally finite quasivariety R of unar algebras with two unar operations such that L is isomorphed to $L_V(R)$.

THEOREM 2. *Let L be finite lattice. Then there exist locally finite quasivariety R (K) of unars (groups) such that L is isomorphed to* $\mathrm{L}_V(R)$ $(\mathrm{L}_V(K))$.

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Minimal clones

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Thorough study of minimal clones was initiated by Béla Csákány in 1983. Since then several interesting families of minimal clones have been discovered, but complete description has been obtained only in particular cases. In this survey talk we shall discuss both kinds of results due to Csákány, Dudek, Jezek, Kearnes, Lengvárszky, Lévai, Machida, Pálfy, Quackenbush, Rosenberg, Szczepara, Szendrei, Waldhauser, and others.

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A structure theorem for dismantlable lattices and enumeration

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The concept of *adjunct* operation of two lattices with respect to a pair of elements is introduced. A structure theorem namely, *A finite lattice is dismantlable if and only if it is an adjunct of chains* is obtained. Further it is established that for any adjunct representation of a dismantlable lattice the number of chains as well as the number of times a pair of elements occurs remains the same. If a dismantlable lattice L has n elements and $n+1$ edges then it is proved that the number of irreducible elements of L lies between $n-2k-2$ and $n-2$. These results are used to enumerate the class of lattices with exactly two reducible elements, the class of lattices with n elements and upto $n+1$ edges, and their subclasses of distributive lattices and modular lattices.

Joint work with N. K. Thakare (University of Pune), B. N. Waphare (University of Pune).

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The Elementary Equivalence of Derived Structures of Free Algebras

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We study the relation on the derived structures (the lattice of subalgebras, the lattice of congruences, the group of automorphisms) of free algebras of some varieties as the relation on the powers of free generators of these algebras.

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Refinement properties in congruence lattices of lattices

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We consider the following general problem: For a CD-variety V, characterize lattices isomorphic to congruence lattices of algebras in V. Recent investigations show that various refinement properties come into play. We present several kinds of such refinement properties and discuss their roles for some lattice varieties.

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The lattice of precompact topologies on $\mathbb{F}_2^{\omega_0}$

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We will discuss the properties of the lattice of precompact ring topologies on the ring $\mathbb{F}_2^{\omega_0}$. The main result of the communication is:

Theorem. *The ring* $\mathbb{F}_2^{\omega_0}$ *has* $\exp\exp\exp\aleph_0$ *precompact ring topologies.*

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Relational equations, relational varieties and minimal relational clones

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The talk presents results investigated mainly by J.-U. Grabowski. Relational clones are closed w.r.t. primitive positive formulas. In analogy to universal algebras (where term operations are given by the clone of operations of an algebra), this gives rise to the notion of "term relation" as well as "relational equation".

In the talk we discuss equationally defined classes of relational systems and relational varieties (= classes closed under retracts and products). The property of having a minimal relational clone can be expressed by relational equations. An equational bases can be given explicitly for all but one classes of relational systems which correspond to the Rosenberg-classes of algebras with maximal clones.

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On the direct product decomposition of lattices

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We prove necessary and sufficient conditions for the direct decomposition of a bounded lattice into the product of directly indecomposable lattices. In the case of a complete lattice L, these conditions can be replaced with equivalent conditions on the set of central-prime elements of L, or with other equivalent conditions concerning the centre of the tolerance lattice of L. We prove that a complete lattice L is a direct product of directly indecomposable lattices if and only if it can be represented as a concept lattice of a clarified context K having the property that any direct product decomposition $L = L_1 \times L_2$ induces a decomposition of the context K into a direct sum $K = K_1 + K_2$ such that $L = L(K_1) \times L(K_2)$.

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On the semigroup of intermediate growth, defined by Mealy automata over a two-symbol alphabet

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It is joint work with prof. V.I. Sushchansky. For definitions see [1], [2]. One of the most interesting problems concerning semigroups, defined by non-initial Mealy automata, is the research of their growth.

The function $\gamma_{\mathfrak{S}}(n)$, $n \in \mathbb{N}$, that at the point n equals to the amount of those elements of the semigroup $\mathfrak{S}$ that can be represented as a product of the length less or equal to n of generators from S is called *the growth function of the semigroup (group)* $\mathfrak{S}$ *relatively to the system of generators* S. A function $\gamma : \mathbb{N} \to \mathbb{N}$ is said to have intermediate growth if for all $d > 0$ there exist numbers $C_1, C_2, N \in \mathbb{N}$ such that $n^d < \gamma(C_1 n) < e^{(C_2 n)}$ for any $n \geq N$.

Let $\mathfrak{A}$ be an automaton shown on fig. 1, and let it defines the automatic transformation semigroup $\mathfrak{S}_{\mathfrak{A}}$. Denote the automatic transformation defined by the automaton $\mathfrak{A}$ at the state q_i as f_i, $i = 0, 1$, and let $R_{p,m}$ be the amount of all partitions of a natural number p to the m positive items.

Theorem. [3], [4]

[1] The semigroup $\mathfrak{S}_{\mathfrak{A}}$ *is a monoid, and has the following co-presentation:*

$$\mathfrak{S}_{\mathfrak{A}} = \langle\ f_0, f_1 \ \mid\ f_0^2 = 1;\ f_1 (f_0 f_1)^n (f_1 f_0)^n f_1^2 = f_1 (f_0 f_1)^n (f_1 f_0)^n, n \geq 0\ \rangle.$$

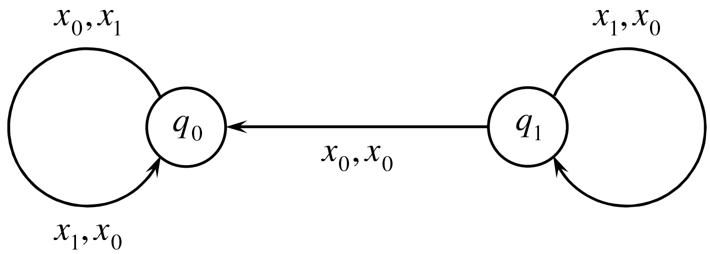

Figure 1: The automaton $\mathfrak{A}$ of intermediate growth

[2] The growth function $\gamma_{\mathfrak{A}}$ *of the automaton* $\mathfrak{A}$ *is defined by equality:*

$$\gamma_{\mathfrak{A}}(n) = (n+2) + \sum_{l=1}^{n-1} \sum_{m=1}^{[\sqrt{l}]} \sum_{p=m}^{d(l,m)} R_{p,m},$$

where $d(l,m) = \left[\frac{1}{2}\left((l+1) - (m-1)^2\right)\right]$, $n \geq 1$.

[3] The growth function $\gamma_{\mathfrak{S}_{\mathfrak{A}}}$ *of the semigroup* $\mathfrak{S}_{\mathfrak{A}}$ *is defined by equality:*

$$\gamma_{\mathfrak{S}_{\mathfrak{A}}}(n) = (2n+1) + 2\sum_{l=1}^{n-2} \sum_{m=1}^{[\sqrt{l}]} \sum_{p=m}^{d(l,m)} R_{p,m} + \sum_{m=1}^{[\sqrt{n-1}]} \sum_{p=m}^{d(n-1,m)} R_{p,m}, \quad n \geq 1.$$

Corollary. *The growth function* $\gamma_{\mathfrak{S}_{\mathfrak{A}}}$ *of the semigroup* $\mathfrak{S}_{\mathfrak{A}}$ *has intermediate growth, and the following inequalities hold:*

$$\left[e^{\sqrt[4]{n}}\right] \leq [\gamma_{\mathfrak{S}_{\mathfrak{A}}}] \leq \left[e^{\sqrt{n}}\right],$$

where $[\gamma]$ *denotes the growth order of* γ.

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Local completeness and locally maximal clones

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Local clones on an infinite universe are defined by a local condition and coincide with the clones preserving sets of finitary relations on A; e.g. subuniverses, congruences and endomorphisms. To classify them we seek large local clones, preferably the maximal ones which are the maximal elements of the poset of local clones distinct from the clone O of all finitary operations on A, ordered by set inclusion. To find them certain techniques from the finite case have been extended yielding e.g. that the clone of all operations isotone with respect to an up- and down-directed order is a maximal local clone. However, we must search through four new sets of locally bounded relations: 1) graphs, 2) reflexive digraphs of diameter 2, 3) binary relations whose transitive hull is a strict order and 4) special ternary relations. In 1) large cliques play an important role, 3) has been quite narrowed while for 2) and 4) we have only partial results. Finally the search among the totally reflexive and symmetric relations has not been completed but it has already yielded many towers; i.e. increasing chains of local clones whose union is contained in no local clone distinct from O.

Joint work with Dietmar Schweigert.

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Ultraproducts and Their Topological Structure

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As was recognised in [1], ultraproducts preserving the validity of certain higher order formulas can be characterized in terms of some topological spaces (which are called ultratopologies). Ultratopologies provide a natural extra structure for ultraproducts and studying this extra structure is useful just for purely model theoretic and algebraic reasons. We will illustrate this by presenting some recently obtained connection between topological and model theoretical properties of ultraproducts. In addition, some results about automorphism groups of finite structures will also be established.

References

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The mystery of the poisoned chocolate bar

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In his book "Combinatorial Games" (Polygon, Szeged, 1998) Béla Csákány decribes the covering game of Gale (GNIM) on the following way: "Frame an $n \times k$ rectangle on a foolscap. We cross out the square on the south-west corner, (0,0), of the rectangle. Two players altenately cross out an empty square with all the squares that can be reached moving only east and north. The one who makes the last move wins."

The game is also known as the divisor game. He describes the strategy for the $2 \times n$ and the $n \times n$ size boards. Surprisingly for no other cases of this 50 year old game is a winning strategy known. The only other (easy) result is that the first player has a winning strategy. We investigate the $3 \times n$ case.

Joint work with Ildikó Molnár-Sáska.

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Equational theories with no sublinear equational bound

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The finite algebra membership problem of a given finite algebra F asks about the membership of any finite algebra in the variety generated by F. We may answer this decidable question via checking the equations true in F. The equational bound of the equational theory of F, as introduced by G. McNulty, tells us the length of equations we have to check in order to decide the membership of an input algebra of less than n elements. If F is finitely based then, obviously, we have a constant equational bound. We set up a sublinear lower bound on the equational bounds belonging to some small algebras built from a few well-known finite algebras, as the first nonfinitely based algebra discovered by Lyndon in 1954, or the algebra used to prove the nondecidability of Tarski's Finite Basis Problem by R. McKenzie in 1996.

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Clones closed under conjugation

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Let G be a permutation group acting on a set A. A clone $\mathcal{C}$ of operations on A is called *G-closed* if for every operation $f(x_1, \ldots, x_n)$ in $\mathcal{C}$, all conjugates $f^g(x_1, \ldots, x_n) = gf(g^{-1}(x_1), \ldots, g^{-1}(x_n))$ of f by permutations $g \in G$ also belong to $\mathcal{C}$. Examples of G-closed clones include Slupecki's and Burle's clones as well as all clones of homogeneous operations. In the talk we will discuss G-closed clones on finite sets for set transitive permutation groups G.

Joint work with Keith A. Kearnes (University of Colorado).

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A Notion of Primality for First Order Structures

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Using *-congruences, N. Weaver introduced the notion of *-variety, which is a class between prevariety and variety; then he established some Mal'cev like conditions for these classes. Following this way, we propose a notion of (quasi)primality for first order structures, and characterize it by suitable notions of simplicity and term interpolation of some functions.

Joint work with Etienne R. Temgoua Alomo (Dept. of Math.; Ecole Normale Supérieure, Univ. of Yaounde-1, Cameroon).

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Congruence classes of small lattice varieties

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The congruence class of a lattice variety is the class of congruence lattices of the members of the variety. We discuss various results and problems related to congruence classes of varieties generated by small non-distributive finite lattices.

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Balanced Semidirect Products and its Endomorphisms

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Let $(M_1, *), (M_2, *)$ - monoids, for which homorphism

$$\rho : M_1 \to \mathfrak{T}(M_2) : t \mapsto \rho^t$$

and antihomomorphism

$$\lambda : M_2 \to \mathfrak{T}(M_2) : t \mapsto \lambda^t$$

are determined ($\mathfrak{T}(X)$ is symmetric semigroup on the set X here). Then the balanced semidirect product $[\rho : M_1 \times M_2 : \lambda]$ of M_1 and M_2, consists of the set $M_1 \times M_2$ equipped product

$$(x;t) * (y;t) = (x * y\lambda^t; t\rho^y * u)$$

The homomorphisms of the balanced semidirect products and its endomorphisms semigroups are described in the terms of the semiretractions that was inroduced in [1].

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Is it difficult to multiply 2 by 2 matrices over the 2 element field?

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In May in Nashville M. Volkov presented a semigroup of size 2^{1700} for which the word problem is co-NP-complete. In our talk we give another example by answering a question of Ross Willard from 1996: The word problem for the semigroup of the 2×2 matrices over the 2 element field is co-NP complete.

Joint work with Csaba Szabó.

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Hasse constants and generalized hexagons

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Hasse constants and their basic properties are introduced to facilitate the connection between the lattice of subalgebras of an algebra C and the natural action of $Aut(C)$ on C. We then work out two examples, and reveal certain combinatorial structures as incidence relations of subalgebras, among others the generalized hexagon of order two.

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Minimal clones with weakly abelian representations

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A characterization of minimal clones with weakly abelian (rectangular, strongly abelian) representations is given. It turns out that if a minimal clone has a nontrivial weakly abelian representation, then it also has a nontrivial abelian representation. Therefore the description follows from Keith Kearnes' results on abelian representations. A nice property of weakly abelian (strongly abelian) representations is that if a minimal clone has a nontrivial weakly abelian (strongly abelian) representation, then all representations are weakly abelian (strongly abelian).

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Parallelogram law and comparability axioms in orthomodular lattices

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Berberian in his book *Baer $*-$rings [1972]* provides monographic and elegant treatment of Baer $*$-rings in which the natural merger of the parts of von Neumann algebra that indicate functional analytic stream and lattice theoretic stream is conspicuous. Berberian's volumnious and detailed exposition mentions several open problems for Baer *-rings. We concetrate on the following two open problems.

Open problem 1: If A is a Baer *-ring with Partial Comparability (PC), does it follow that A has Generalized Comparability (GC)?

Open problem 2: If A is a Baer *-ring with (GC) and if e, f are finite projections in A, is $e \vee f$ finite? In other words, is it true that in a Baer *-ring with (GC) the finite projections form a sublattice of the lattice of all projections of A?

We have studied the comparability axioms in orthomodular lattices with suitable equivalence relation, and obtained equivalence between (GC) and (PC) in a general *-ring imposing some restrictions on the lattice of projections. In the similar way using parallelogram law we have obtained the sublatticeness of finite projections in a *-ring.

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