Tolerance factorable varieties and four ways Béla Csákány influenced their study*

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1. Mal'cev (Maltsev) conditions $\longrightarrow$ 

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G.

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G.Cz.(**1982**): $\mathcal{L} = \{$all lattices$\}$ is strongly tolerance factorable.

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$\sim$

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 , $(1-\lambda)\cdot$  $+\,\lambda\,\cdot$ 

4.

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4. My 1st refereeing task ≈ 1980

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Independent join /product of subvarieties $\mathcal{V}_1, \ldots \mathcal{V}_n$: $t \approx e_n^i$.
(W. Taylor 1975, G. Grätzer, H. Lakser, and J. Płonka 1969).

Exampl

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Example: rectangular bands

Natural decomposition with **no skew**: S, ⊖. We (little step) now skew T, B.

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But now: ∞

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Theorem: Let $\mathcal{V}$ be the independent join of its subvarieties $\mathcal{V}_1, \ldots, \mathcal{V}_n$, with all the $\mathcal{V}_i$ being strongly tolerance factorable. Then $\mathcal{V}$ is strongly tolerance factorable.

Sketch of proof: nothing is skew.

TImC.

TImC. $\mathcal{L}$ (G.Cz. and G. Grätzer, AU 2011).

P

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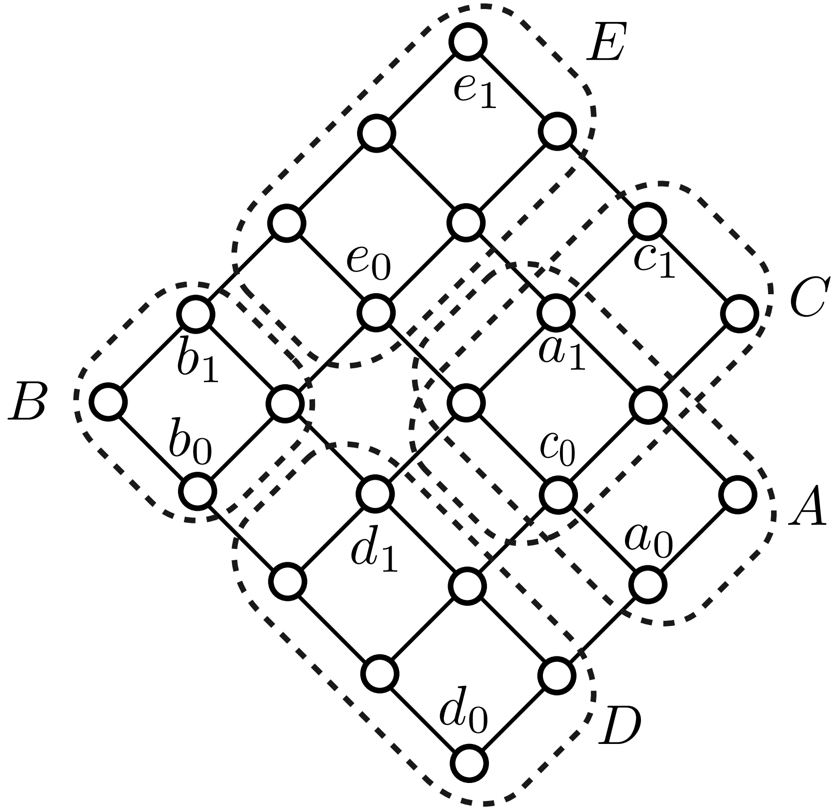
Proposition: Independent join preserves TImC.

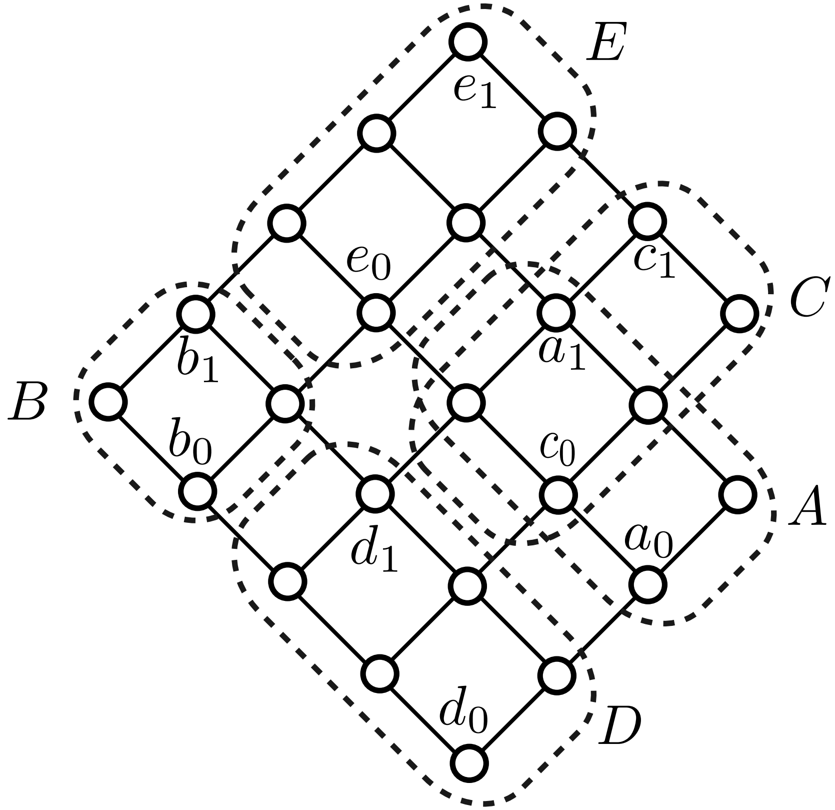
Theorem: Strong tolerance factorability implies TImC.

Theorem (We 3 plus P. Lipparini): $\mathcal{V}$ by linear identities $\Rightarrow$ TImC (definition: next talk).



$\mathcal{L}$. $\mathcal{L}^{(3)}$ (ternary version):
$t_\vee(x,y,z) = x \vee (y \wedge z)$, $t_\wedge(x,y,z) = x \wedge (y \vee z)$,

$x \vee y \mapsto t_\vee(x,y,y)$ and dually: $|\Sigma| = 6 + 2$, the two: $t_\vee(x,y,z) = t_\vee(x, t_\wedge(y,z,z), t_\wedge(y,z,z))$ and its dual. Equivalent *alter ego*.



$\{t_\vee(x,y,z) : x \in A, y \in B, z \in C\} = [c_0, a_1]$. ! $\mathcal{L}_3$ is **not** tol.fact.

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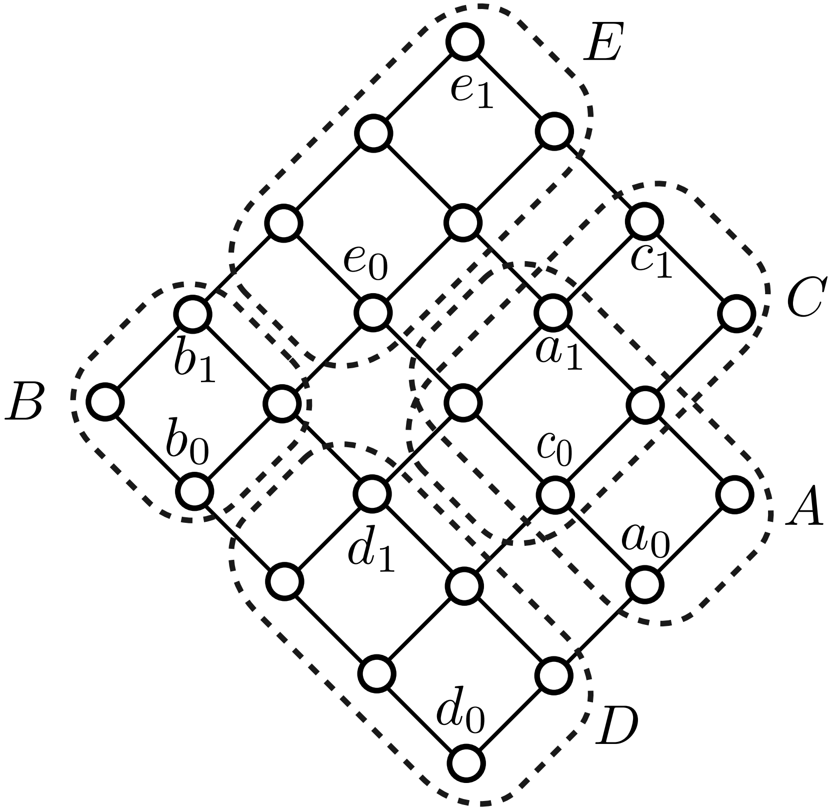

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Open problem: give known $\mathcal{V}$ without (strong) tolerance factorability and an alter ego with (strong) tolerance factorability!

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