

SUDOKU rules and history

Sudoku is a larger square consisting of 9×9 squares. The square is divided into nine smaller 3×3 squares, in which we find some numbers from 1 to 9 scattered around. The players fill in the remaining empty squares with their own numbers (also from 1 to 9) so that in each horizontal row, vertical column, and 3×3 square, the numbers from 1 to 9 appear exactly once.

The basic idea of the game comes from the mathematician Leonard Euler, who lived in Switzerland in the 18th century. He invented what we now call a "Latin square": in an $n \times n$ Latin square, the numbers $1, 2, \dots, n$ each appear exactly once in each row and column. The name comes from Euler's use of Latin letters instead of numbers.

A predecessor of sudoku appeared in a French newspaper in 1892, in which the 9×9 square was already divided into smaller 3×3 squares. It still had multi-digit numbers, and the squares had to be filled according to different rules. The game in its current form was invented by American architect Howard Garns in 1979. The game arrived in Japan in 1984, where it was first published in Nikoli magazine as a puzzle to be solved. The name at that time (Suuji wa dokushin ni kagiru: numbers can only appear once) gave rise to the name sudoku today.

Some basic rules that can speed up the solving of SUDOKU

1. If you have managed to determine a number in a square, then that number is deleted from all other squares in the row, column and block of the square. This can quickly reduce the number of possibilities in other squares.
2. If you have managed to determine the location of a number, then it is worth working on that number further: see if you can fill in the same number elsewhere. It is also useful to write down the numbers that you have already filled in each row/column/block so that you do not have to deal with them again.
3. If you have managed to reduce the number of possibilities in a given square, then it is worth looking at the row/column/block of that square to see if the fewer possibilities allow you to apply one of the methods.
4. It is worth working on those rows/columns/blocks where many squares are already filled in, the rest are usually easier to fill in.
5. Some of the methods below are based on the fact that only a few (a specific number, usually 2–3) possibilities can occur in one or more given squares. It is worth looking for such squares and looking at their relationships according to the method's procedure.
6. Some methods are based on the fact that a given number can only occur in a few (usually 2–3) places in a row/column/block. It is worth looking for such rows/columns/blocks and looking at their relationships according to the method's procedure.

Some methods that can be used to solve SUDOKU

1. ***n*-option** ($n \geq 1$, $2n - 2$ points): If there are n squares in a given row, column or block on which at most n different numbers can occur, then these n numbers must occur in exactly these n squares in some order. So this n number can be deleted from all other squares in the given row, column or block.

2. **n -hidden option ($n \geq 1$, $2n - 1$ points):** If there are n numbers in a given row, column or block that can only occur in n squares, then these n numbers must occur in exactly these n squares in some order. So from the n squares, all other options except the given n number can be deleted.
3. **Locked option (4 points):** If in a row or column S a number n occurs only within a block B , then this number n cannot occur outside the row or column S in the block B , so this number can be deleted from the corresponding squares of the block. In this reasoning, the row/column and the block can be interchanged, so the following method also works. If in a block B a number n occurs only within a row or column S , then this number n cannot occur outside the block B in the row or column S , so this number can be deleted from the corresponding squares of the row or column.
4. **2 row/column X-wing (6 points):** If the rows or columns S_1, S_2 are such that they pass through identical blocks, and there is a number n in them that occurs only in the blocks B_1, B_2 , then in these two rows/columns the number n must occur in both the blocks B_1 and B_2 . So in the two blocks the number n cannot appear outside the given rows/columns S_1, S_2 , so the number can be deleted from the corresponding squares of these blocks. Of course, the X-wing can also be formulated so that only the rows and columns are included, and in this case more than two rows/columns can play a role:
5. **n X-wing ($n \geq 2$, $3n$ points):** If within the rows $S_1, S_2, \dots, S_n$ the number k can only occur in the columns $O_1, O_2, \dots, O_n$, then in this $n \times n$ square part the number k must occur in every row and column exactly once. So in the columns $O_1, O_2, \dots, O_n$ the number k cannot occur outside the rows $S_1, S_2, \dots, S_n$, so the number k can be deleted from the corresponding squares of the columns. Here too, the roles of row and column can be interchanged.

Definition: If a square A is in the same row, column or block as another square B , then I say that *square A can be seen from square B* .

6. **XY-wing (sometimes called Y-wing, 7 points):** If only two numbers (n_1, n_2) can occur as possibilities in square A , only two numbers can occur in square B visible from A , namely n_1, n_3 , and only two numbers can occur in square C visible from A , namely n_2, n_3 , then either the number n_1 or n_2 is in square A , and the number n_3 must be in square B or C . Thus, the number n_3 can be deleted from all squares that are visible from both squares B and C .
7. **XYZ-wing (10 points):** If only three numbers (n_1, n_2, n_3) can occur as possibilities in the A square, only two numbers can occur in the B square visible from A , namely n_1, n_3 , and if only two numbers can occur in the C square visible from A , namely n_2, n_3 , then either the number n_3 must be in the A square, or (according to the previous reasoning) the number n_3 must be in one of the B and C squares. Thus, the number n_3 can be eliminated from all squares that are visible from all A, B , and C squares.
8. **Forced option (points: four times the number of steps):** This is practically not much different from guessing. In a previously unfilled square, we choose a number from the options there, and assume that that number is in that square. We start solving the SUDOKU (usually with the simpler methods), and if we get a contradiction, then the number in the starting square was not the number we entered, that number can be deleted from the options.