KRITIKUS PONTOK KÖVETÉSE IDŐBEN VÁLTOZÓ SIMA FELÜLETEKEN ÉS FINOM DISZKRETIZÁCIÓIKON

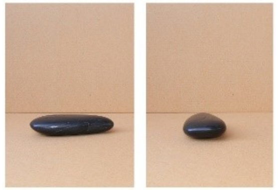
DOMOKOS GÁBOR, LÁNGI ZSOLT ÉS SIPOS ANDRÁS

MTA-BME MOROFDINAMIKA KUTATÓCSOPORT

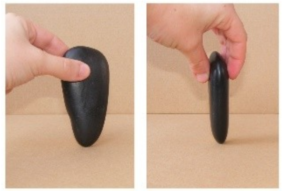
SZEGEDI GEOMETRIAI NAP, 2017. OKTÓBER 6.

GEOPHYSICAL MOTIVATION: FLUVIAL ABRASION AND EQUILIBRIA

HAND EXPERIMENTS WIT PEBBLES:

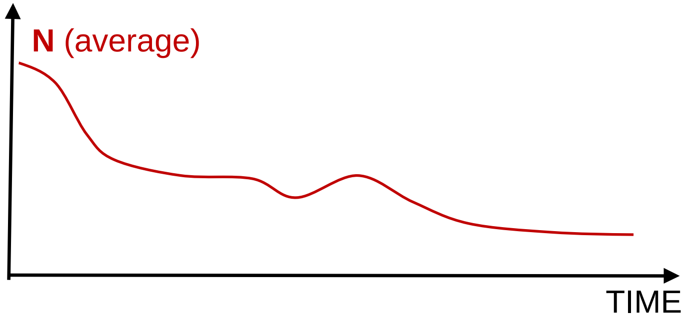


STABLE (S)



UNSTABLE (U)

FIELD OBSERVATION:



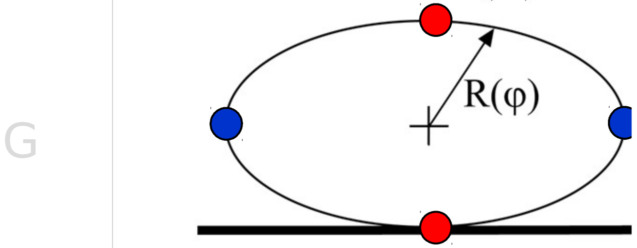
STATIC EQUILIBRIA

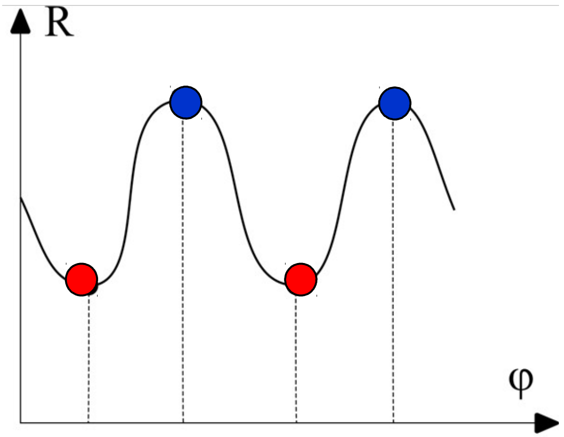
a) 2D: S=U

N=S+U

STABLE (S)

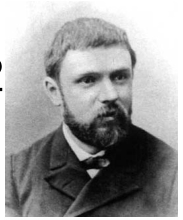
UNSTABLE (U)





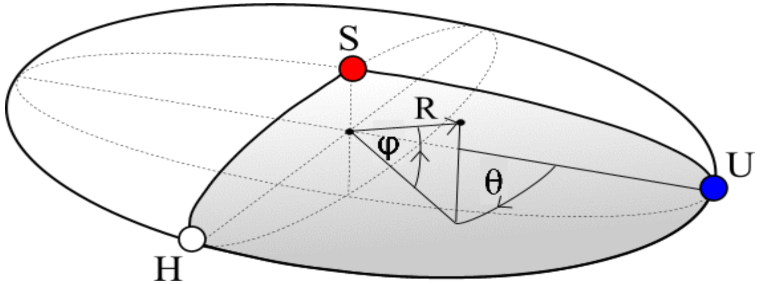
b) 3D: S+U-H=2

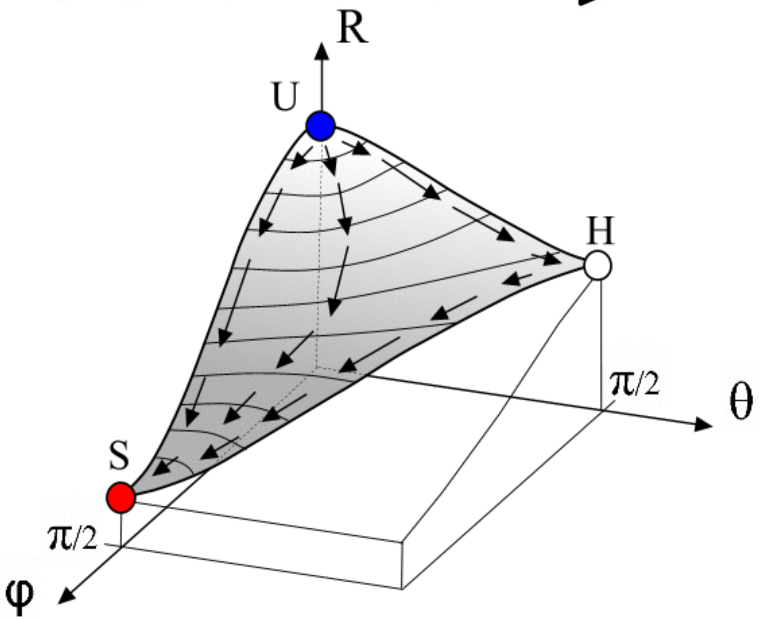
N=S+U+H



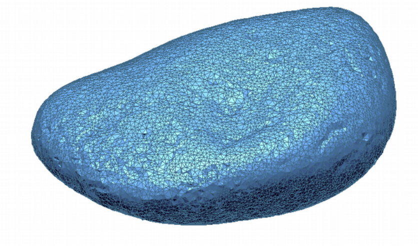
Jules Henri Poincaré (1854-

Eberhard Friedrich Hopf (1902-1983)



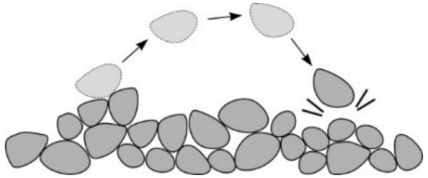


Fluvial abrasion of pebbles: intuiti



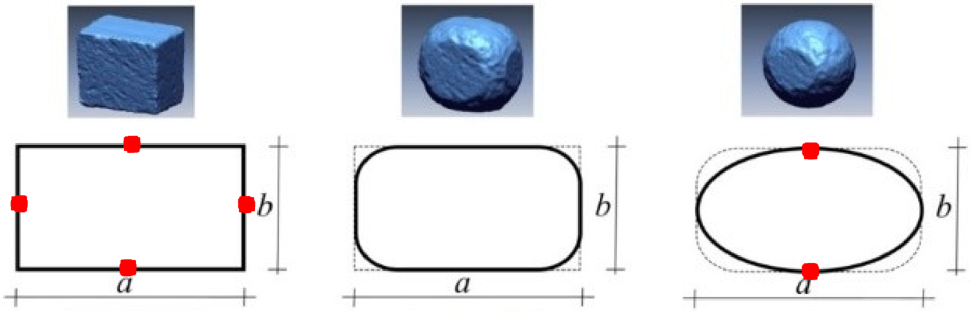
Field observations

Abrasion by many collisions



Continuum approach =global model)

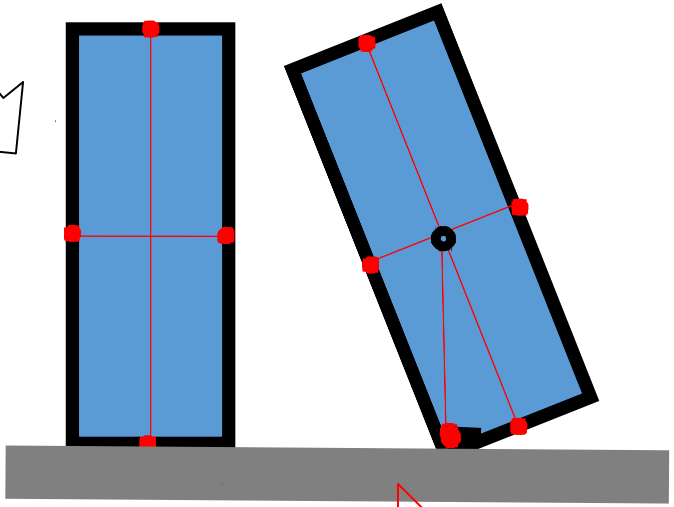
Discrete approach =local model)



S= 4

EVOLUTION

S= 2



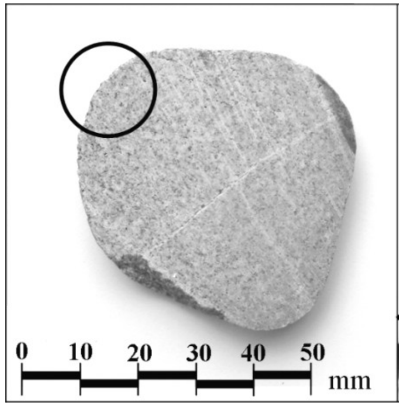
S= 4

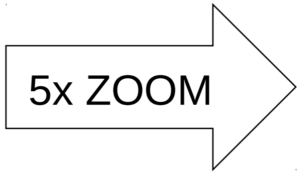
EVOLUTION

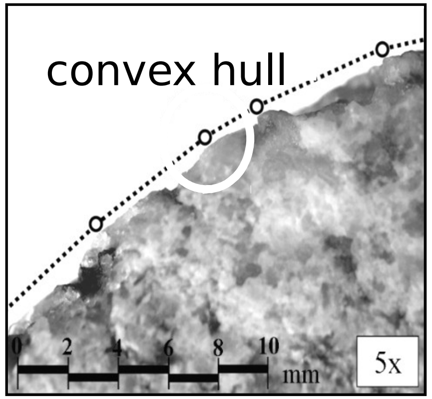
S= 5

GLOBAL AND LOCAL MODELS OF PEBBLE CONTOUR

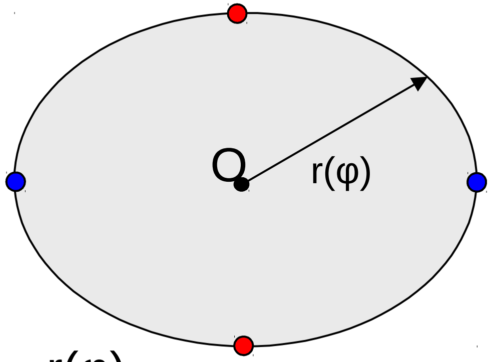
PEBBLE CONTOUR:



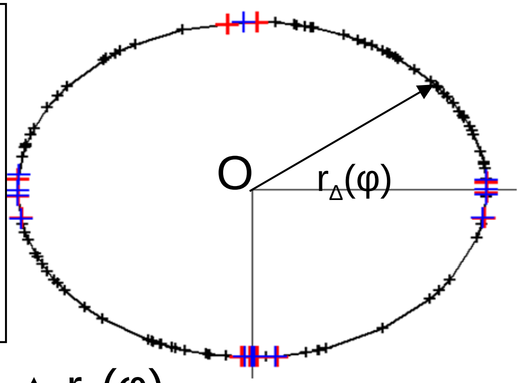


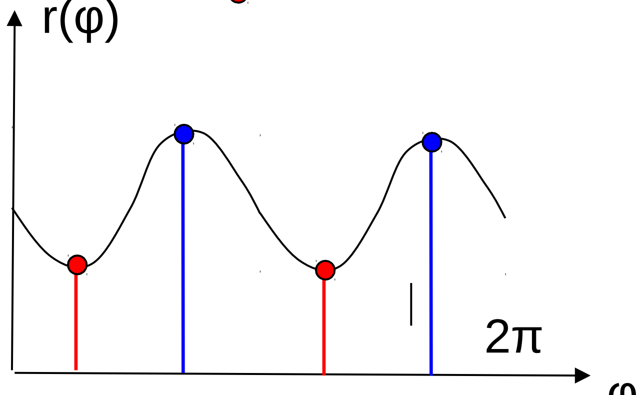


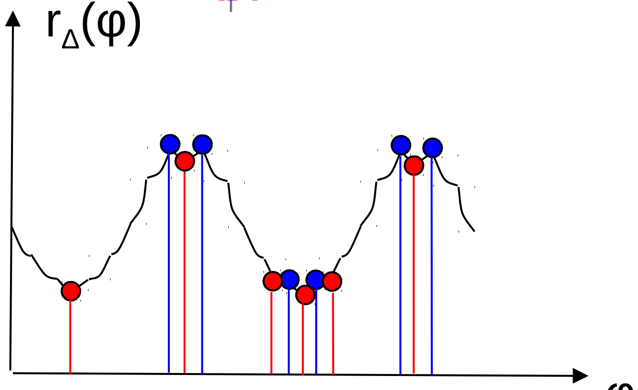
GEOMETRIC MODEL: GLOBAL: C^3-smooth curve $r(\varphi)$ LOCAL: Polygon $r_\Delta(\varphi)$



	EQUILIBRIA	
	Stable	
S=2		S=9
	Unstable	
U=2		U=9
	Total	
N=4		**N_Δ=18**

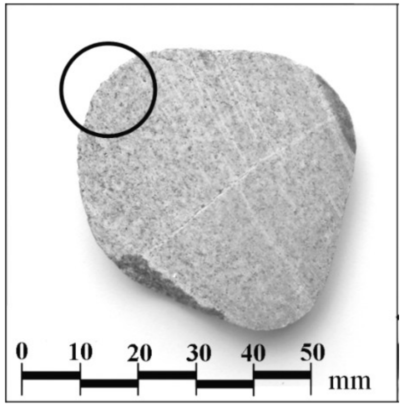




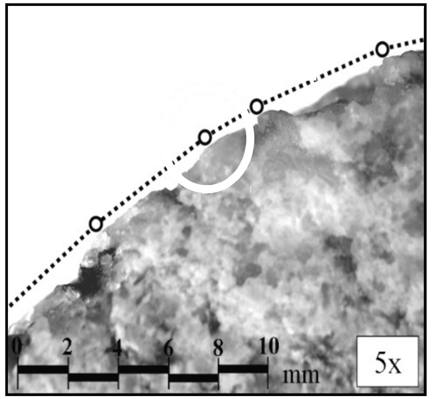


GLOBAL AND LOCAL MODELS OF PEBBLE CONTOUR

PEBBLE CONTOUR:



5x ZOOM



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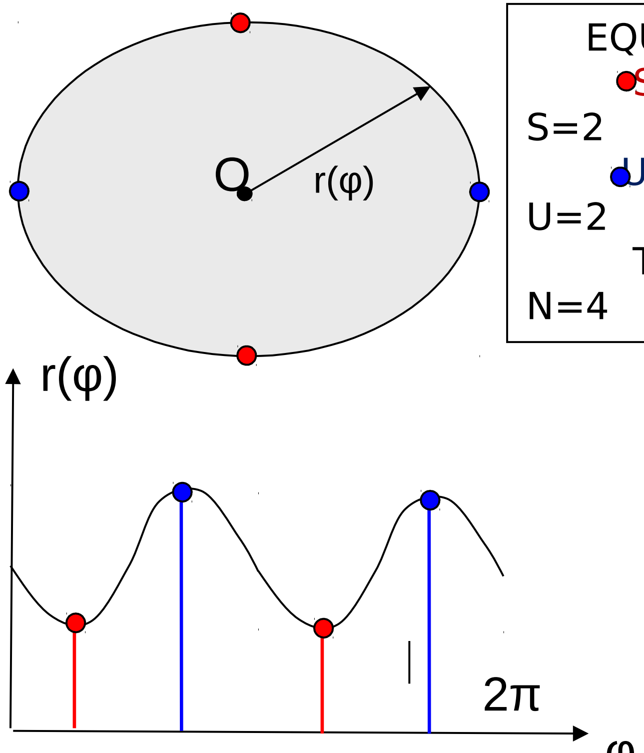
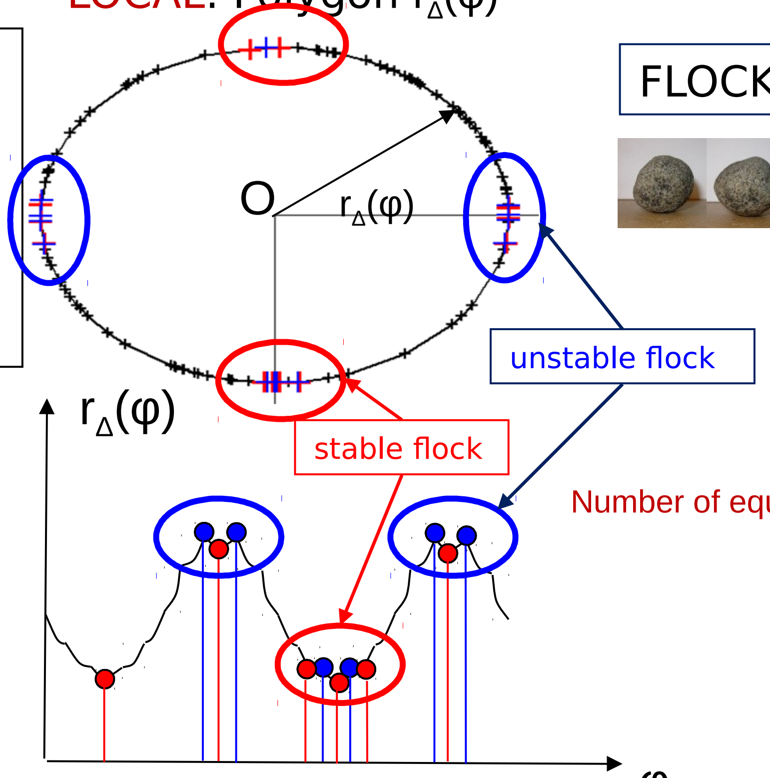


EQUILIBRIA

Stable	
S=2	S=9
Unstable	
U=2	U=9
Total	
N=4	N_Δ=18



FLOCKS

Number of equilibria in i^{th} flock = N_Δ^i

(A) „STATIC” THEORY: GENERIC FLOCKS

ASSUMPTIONS:

1. $r(\varphi)$ is C^3-smooth
2. Discretization is uniform
3. Equilibrium is generic, i.e. $dr/d\varphi=0$, $d^2r/d\varphi^2 \neq 0$

RESULTS:

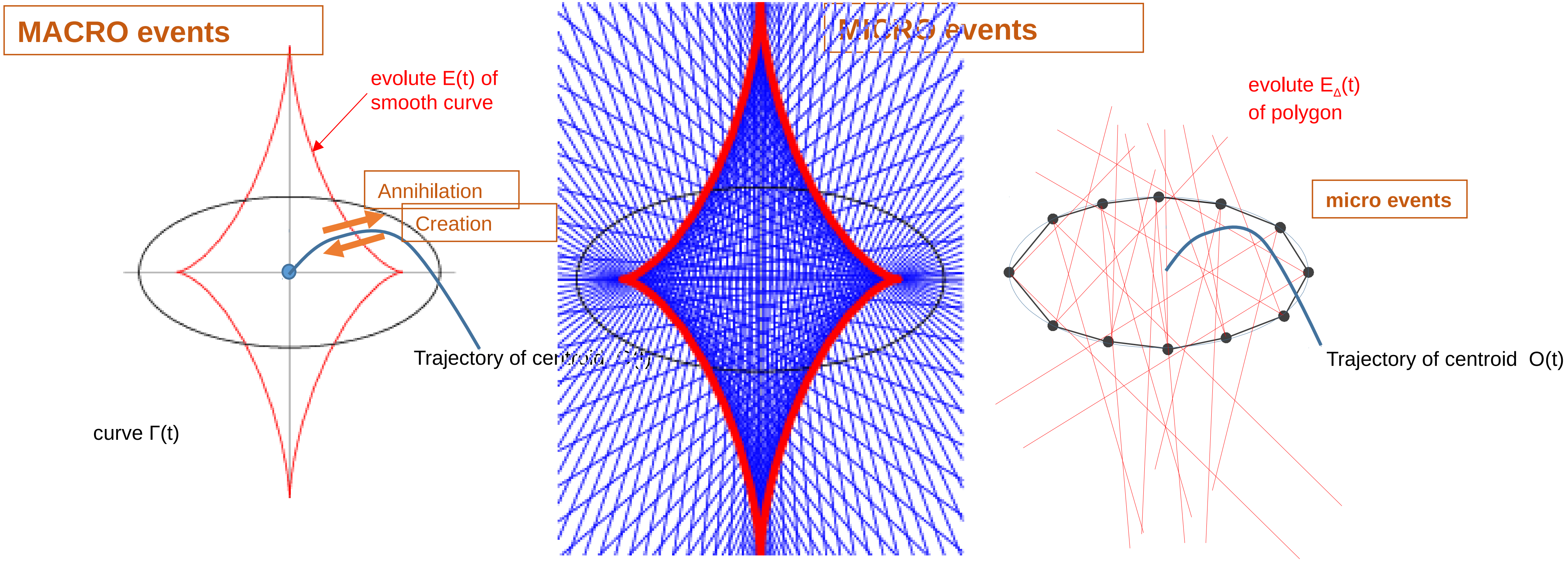
1. GENERIC FLOCK SIZE $\lim_{\Delta\to 0} (N_\Delta^i)=(1+|r\kappa|)/|1+r\kappa|$, where κ is the curvature (Monatshefte)
2. NO EQUILIBRIA OUTSIDE FLOCKS: $\lim_{\Delta\to 0} N_\Delta = \sum_i \lim_{\Delta\to 0} (N_\Delta^i)$ (THEOREM 1)
3. SMALL RANDOM FLUCTUATIONS OF Δ CAUSE SMALL CHANGES IN N_Δ (THEOREM 2)

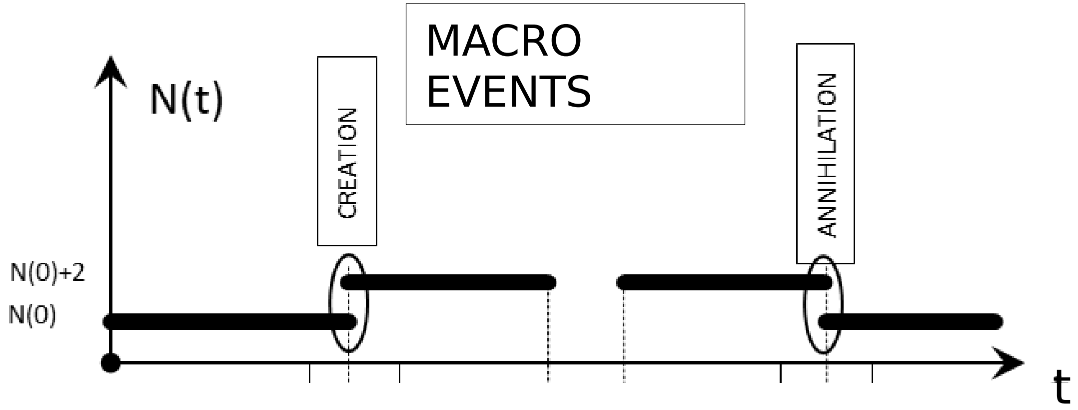
(B) „DYNAMIC” THEORY: $N(t)$, $N_\Delta(t)$ AND CRITICAL FLOCKS

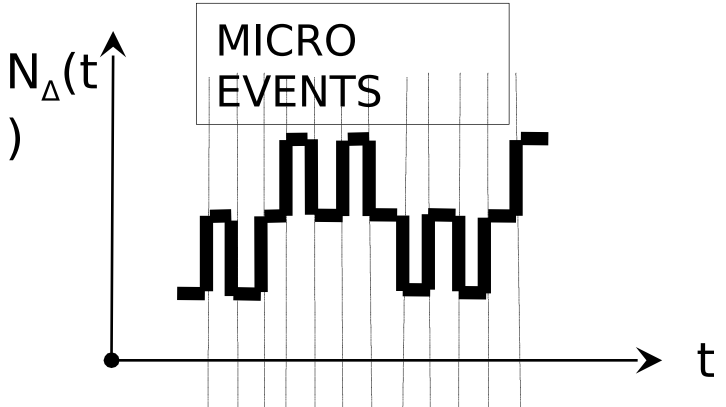
ASSUMPTIONS:

1. $r(\varphi,t)$ is C^3-smooth
2. Discretization is uniform for all t

(B) „DYNAMIC” THEORY: GEOMETRIC INTERPRETATION AND TYPE OF EVENTS







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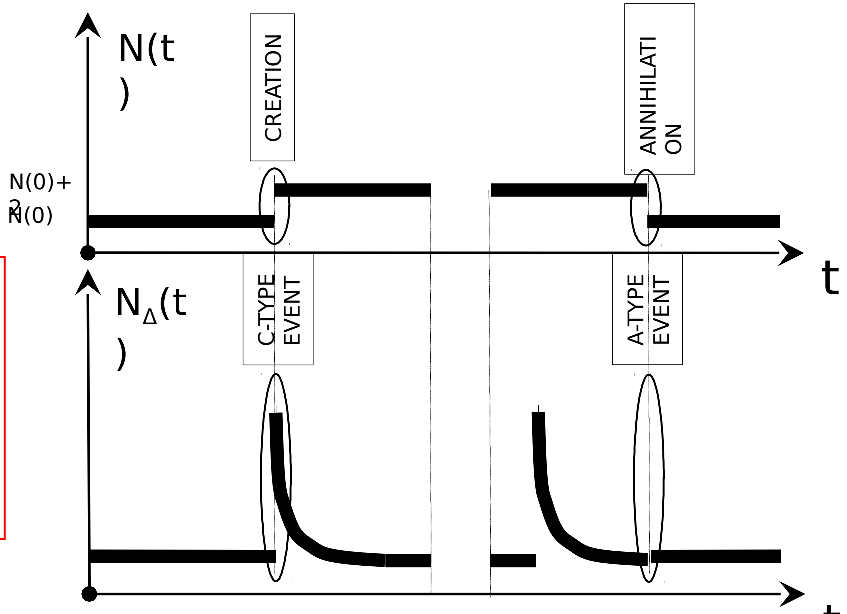
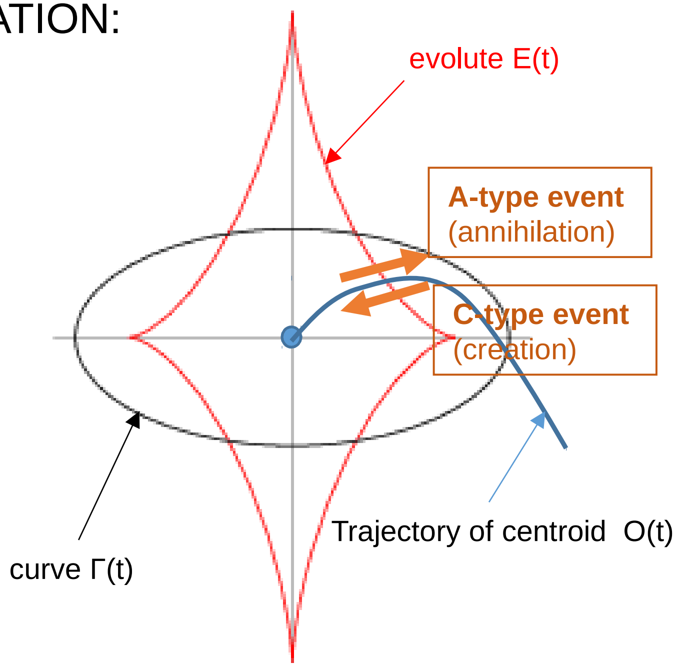


Theorem: (THEOREM 3)

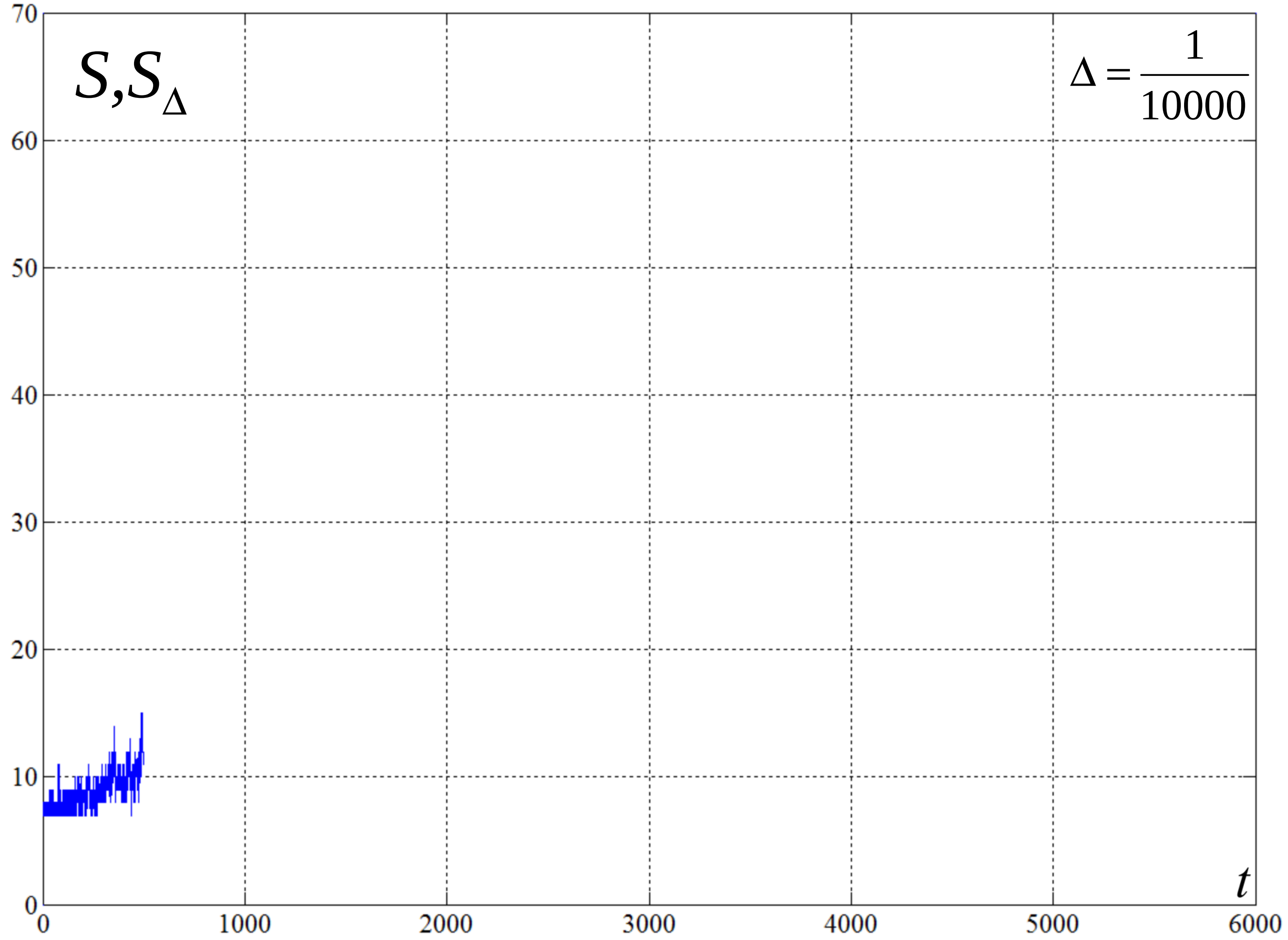
$N(t)=N(0)+2(c-a)$

Where *c* and *a* refer to the number of C- and A-type events, respectively.

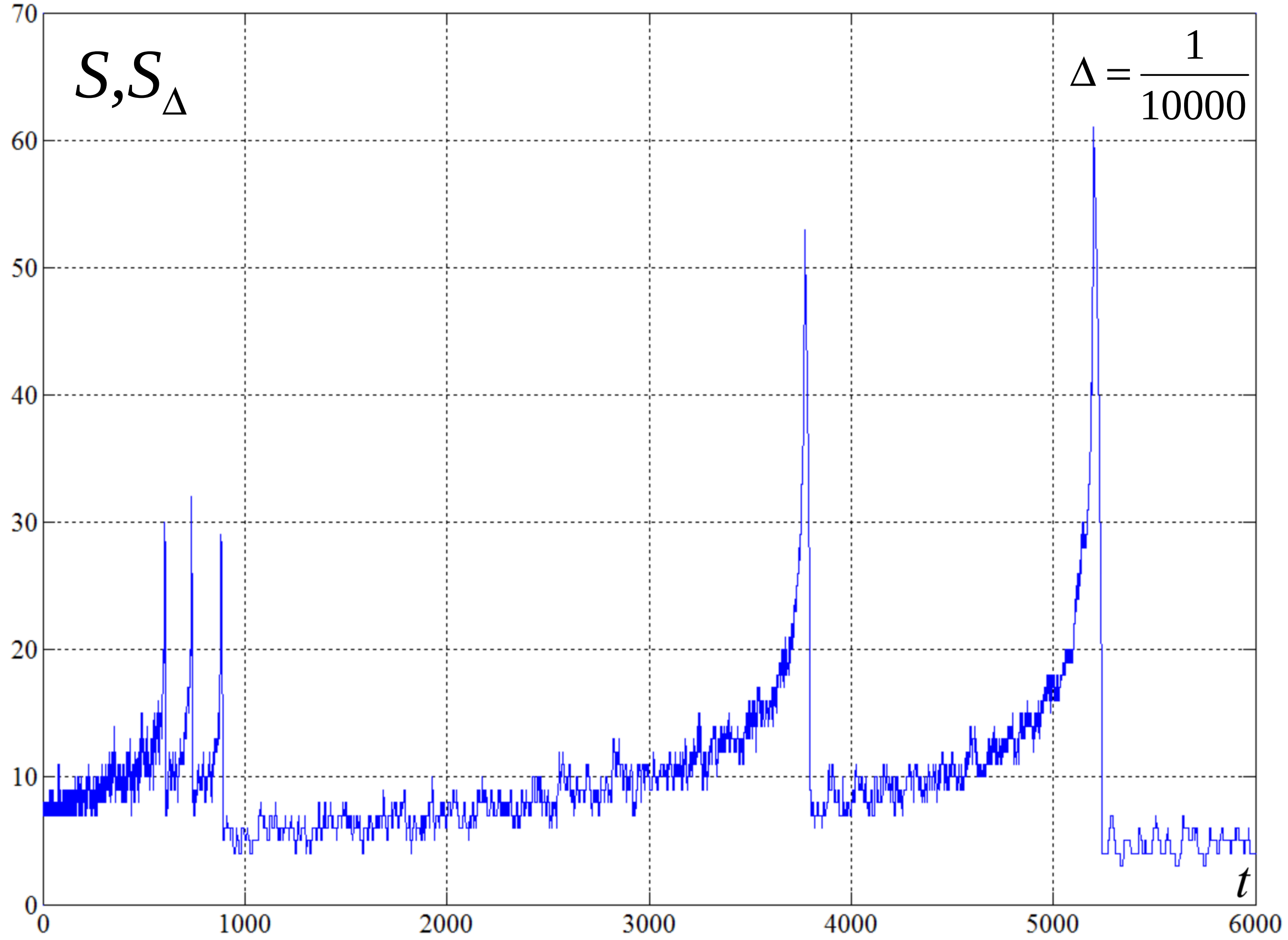
INTERPRETATION:



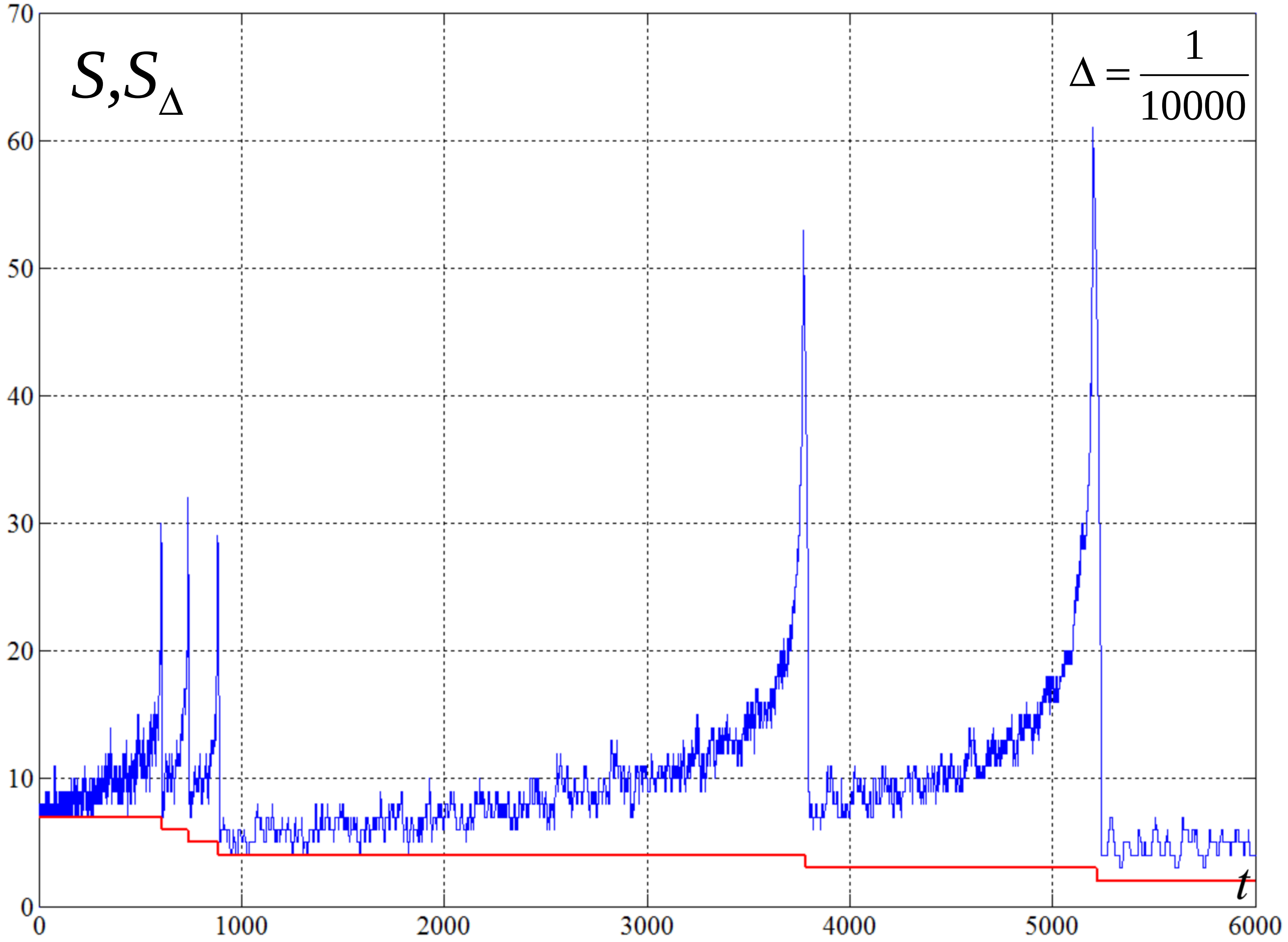
Evolution of equilibria in 2D



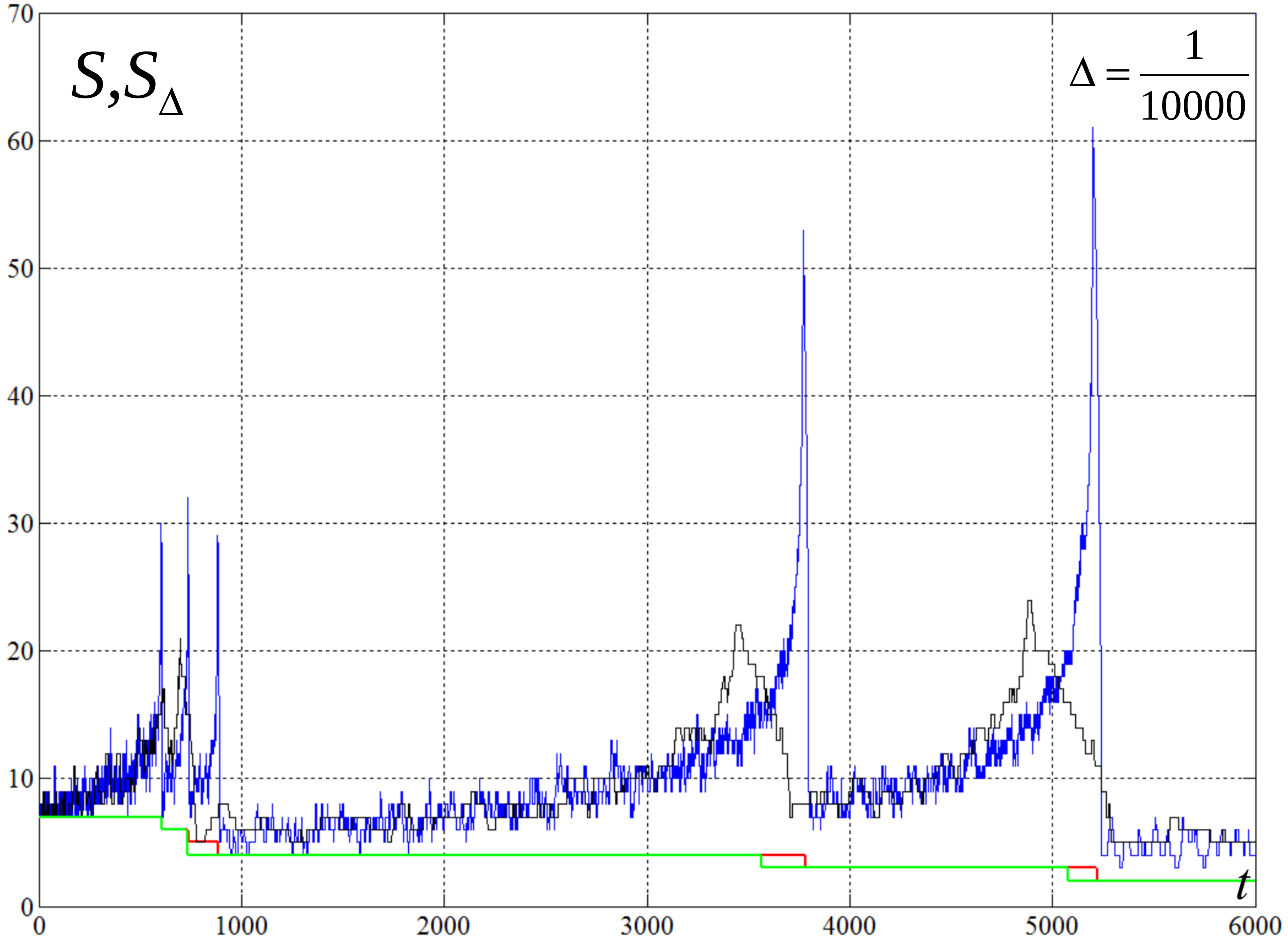
Evolution of equilibria in 2D



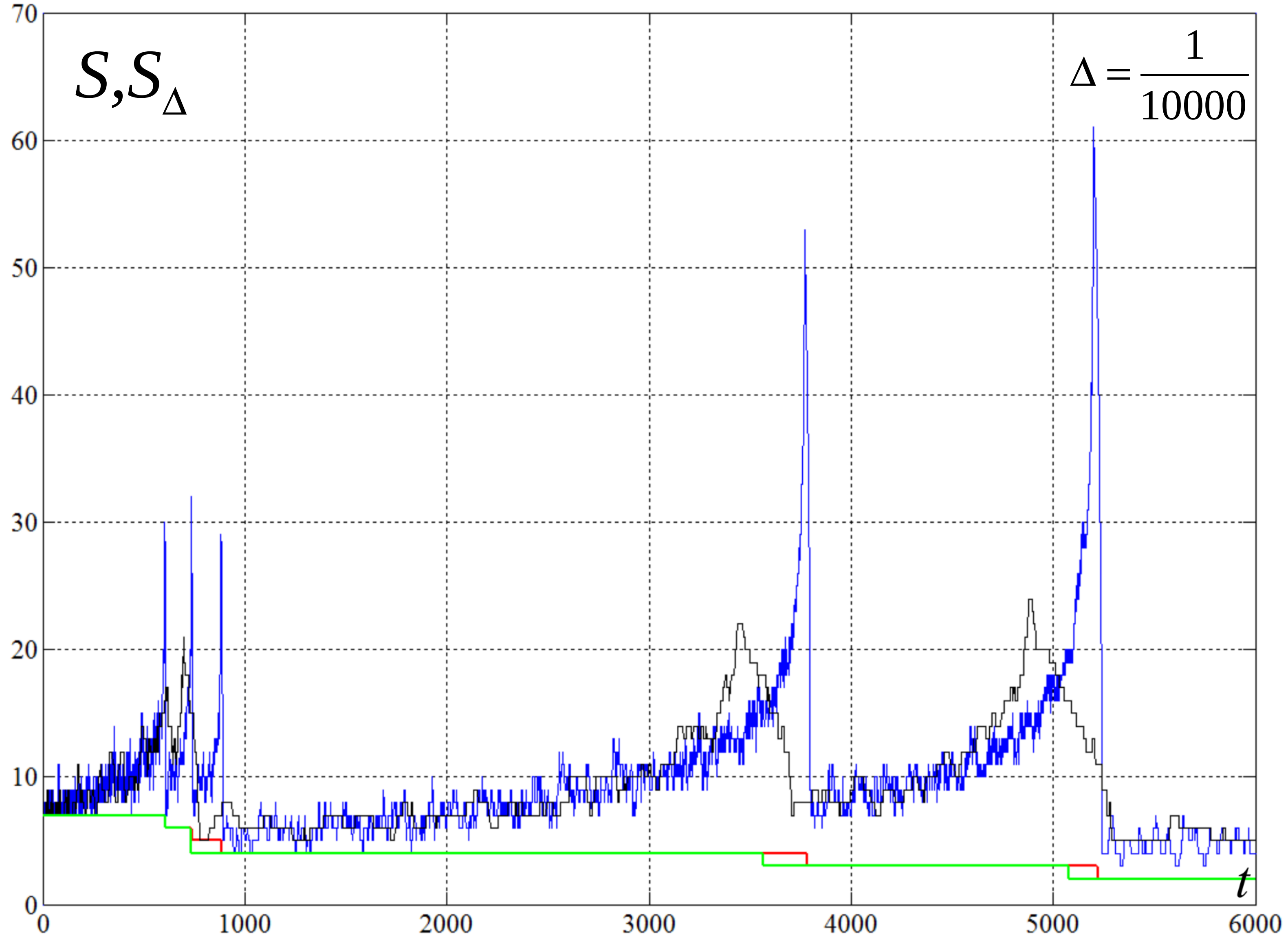
Evolution of equilibiria in 2D



Evolution of equilibria in 2D



Evolution of equilibria in 2D



MORE RESULTS ON CRITICAL FLOCKS

1. **SPATIAL EVOLUTION (DIAMETER) AT FIXED, CRITICAL TIME t=t*.** IF THE FIRST NON-VANISHING TERM IN THE TAYLOR SERIES OF *r(φ)* IS OF ORDER *k* THEN THE DIAMETER *D* OF THE FLOCK CAN BE WRITTEN AS

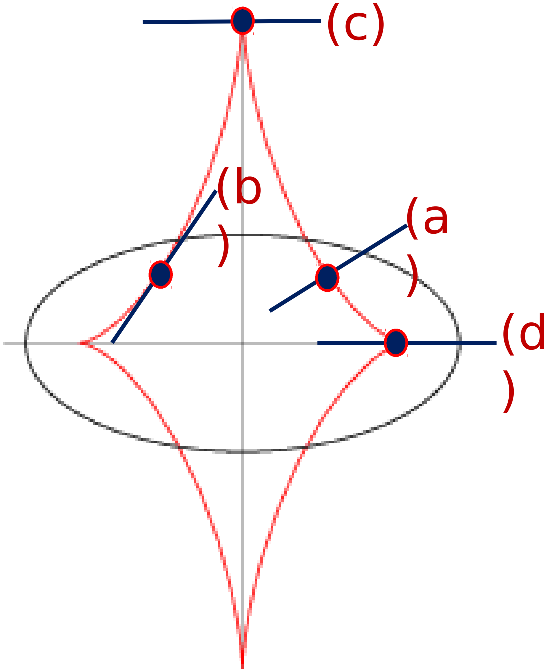
   $D = \Theta(\Delta^{1/(k-1)})$ (THEOREM 4)

Corollaries:

a) For generic (codimension 0) flocks we have $k=2$, thus $D = \Theta(\Delta)$

b) At codimension 1, generic saddle-node bifurcations we have $k=3$, thus $D = \Theta(\Delta^{1/2})$

c) At codimension 2 bifurcations (e.g. generic cusps) we have $k=4$, thus $D = \Theta(\Delta^{1/3})$



2. **TIME EVOLUTION AT FIXED, SUFFICIENTLY SMALL Δ.** IF THE INTERSECTION OF O(t) AND THE EVOLUTE E(t) AT t=t* IS

a) transversal at a generic point of the evolute then, as t -> t* we have $N_\Delta(t) = \Theta(t^{-1/2})$ *on one side, and* $N_\Delta(t) = const.$ *on the other side,*

b) tangential at a generic point of the evolute then, as t -> t* we have $N_\Delta(t) = \Theta(1/t)$,

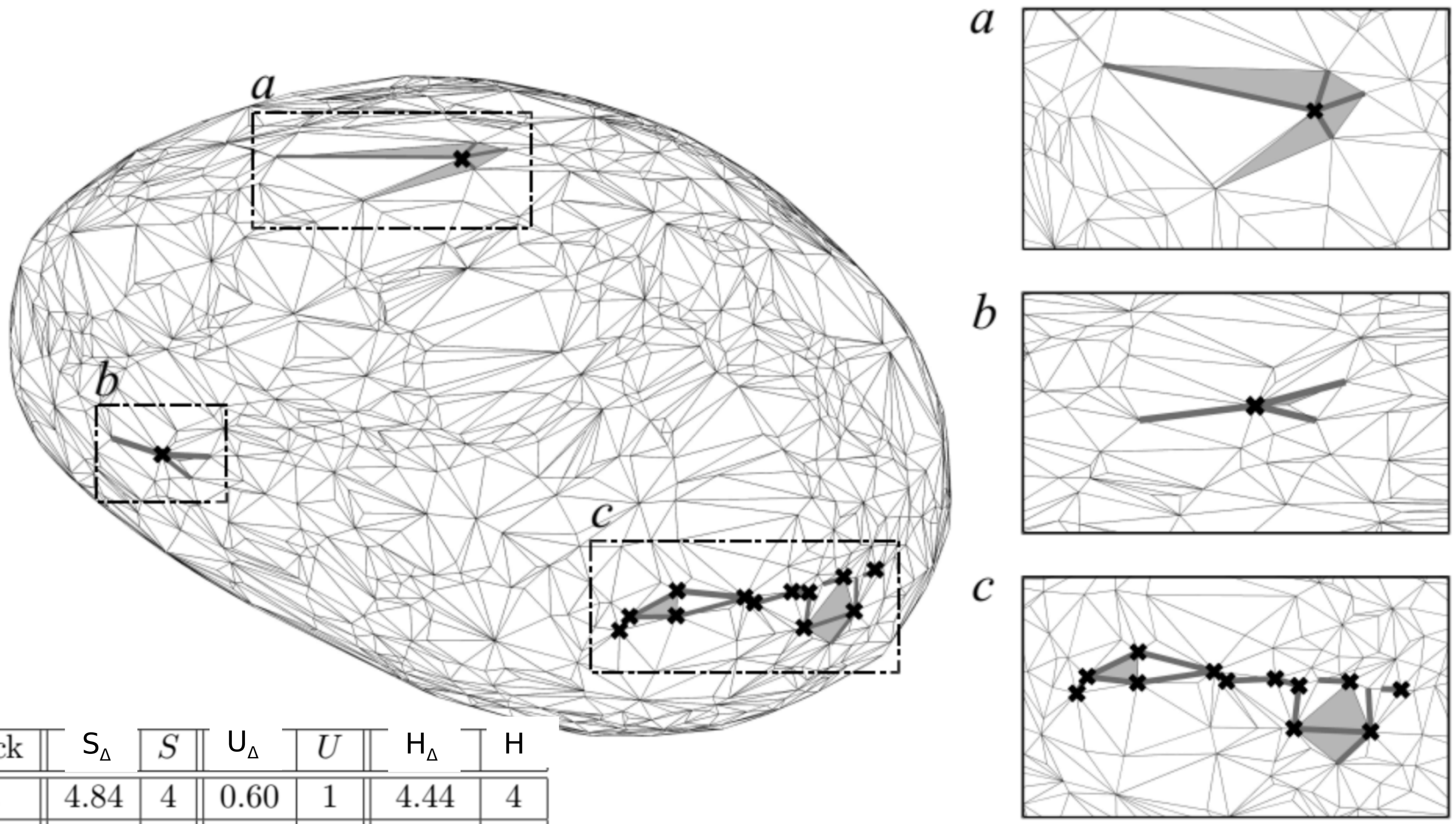
c) transversal at a generic cusp point of the evolute then, as t -> t* we have $N_\Delta(t) = \Theta(t^{-2/3})$,

d) tangential at a generic cusp point of the evolute then, as t -> t* we have $N_\Delta(t) = \Theta(1/t)$.
(THEOREM 5)

SOME ILLUSTRATIONS IN 3D

1. SIZE OF GENERIC FLOCKS:

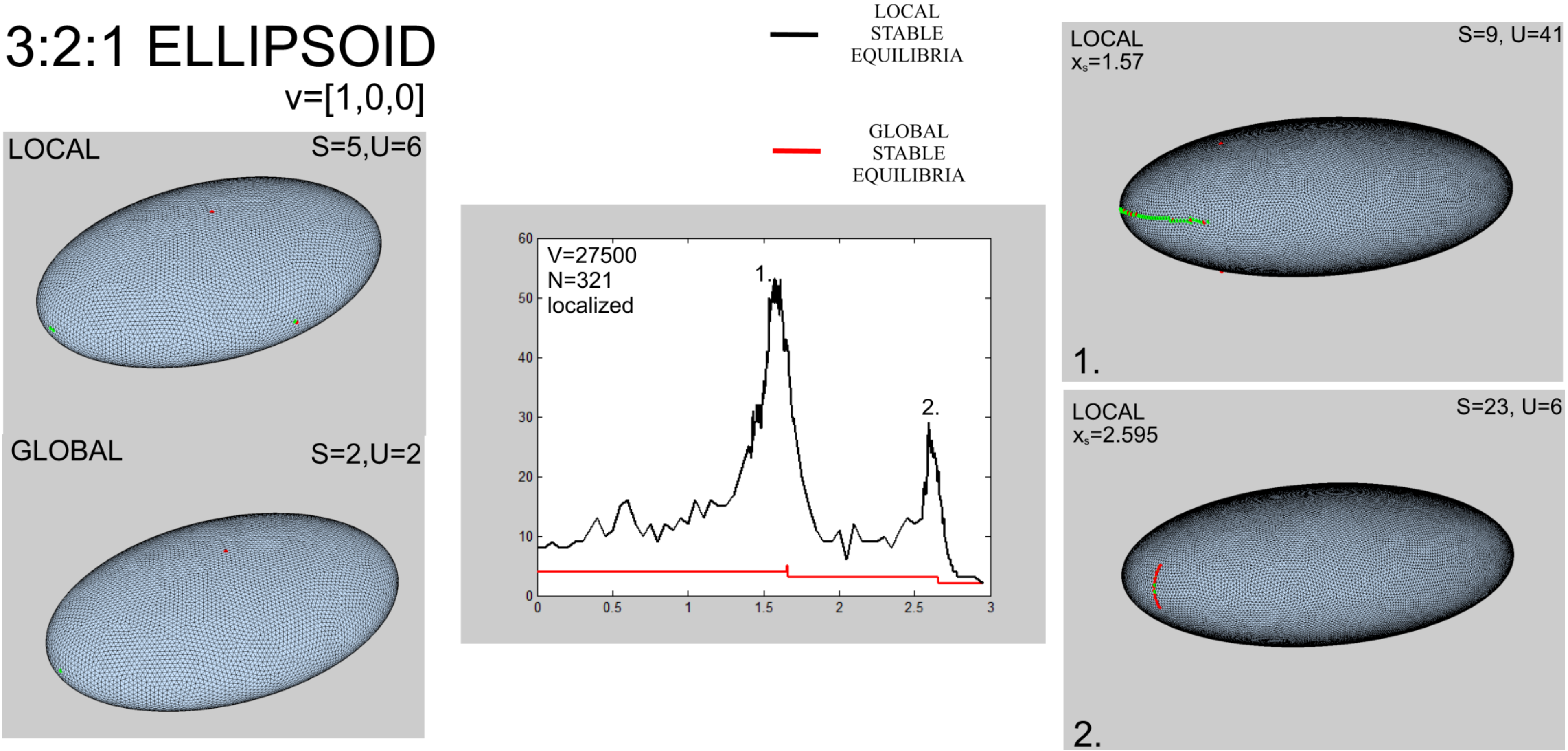
$S_\Delta{}^i = d^i$, $U_\Delta{}^i = \kappa_1{}^i \kappa_2{}^i (r^i)^2 d^i$, $H_\Delta{}^i = -(\kappa_1{}^i + \kappa_2{}^i) r^i d^i$, where $1/d^i = |(1+\kappa_1{}^i r^i)(1+\kappa_2{}^i r^i)|$ (MONATSHEFTE)



flock	S_Δ	S	U_Δ	U	H_Δ	H
a	4.84	4	0.60	1	4.44	4
b	0.72	1	1.03	1	2.75	3
c	2.88	3	9.55	12	11.43	14

2. CRITICAL FLOCKS ON TRI-AXIAL ELLIPSOID

Here we move the reference point (=centroid) along the major axis of the ellipsoid. Observe the two peaks in the number of local equilibria as the reference point crosses the two evolutes.



2. DEGENERATE, UMBILIC FLOCKS ON TRI-AXIAL ELLIPSOID

Here we move the reference point (=centroid) approximately in the direction of the origin of the osculating sphere at the umbilic point of the ellipsoid..

2:1.5:1 ELLIPSOID

v=[1,0,a/c*[(b*b-c*c)/(a*a-b*b)]^3/2]