Iterative Optimization in Quantum Metrology and Entanglement Theory Using Semidefinite Programming

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Quantum metrology [1] is about measuring physical parameters with precision beyond classical limits by exploiting quantum effects. The metrological performance of a quantum state is measured by how much it can outperform all separable states in a metrological task. We present efficient optimization techniques to maximize this performance by searching for the optimal local Hamiltonian generating the unitary dynamics for a given bipartite initial state [2]. We show that this is equivalent to maximizing the Quantum Fisher Information over a specific set of local Hamiltonians. This task is highly non-trivial, as it involves maximizing a convex function over a convex set. We reformulate the problem in a bilinear way and optimize it using an iterative see-saw method, where each optimization step is solved via semidefinite programming. This way we obtain a lower bound to the maximally achievable performance. We also apply relaxation methods that give upper bounds for the optimum.

We further show that the same optimization framework can be adapted to problems in entanglement theory, such as identifying bound entangled states that maximally violate the Computable Cross Norm-Realignment criterion. Finally, we provide examples where two copies of a quantum state outperform a single copy, demonstrating metrological activation [3] for certain small systems.

[1] L. Pezze, A. Smerzi, M. K. Oberthaler, R. Schmied, P. Treutlein, Quantum metrology with nonclassical states of atomic ensembles, *Rev. Mod. Phys.* **90** (2018), 035005.

[2] Á. Lukács, R. Trényi, T. Vértesi, G. Tóth, Iterative optimization in quantum metrology and entanglement theory using semidefinite programming, *Quantum Sci. Technol.* **11** (2026), 015042.

[3] R. Trényi, Á. Lukács, P. Horodecki, R. Horodecki, T. Vértesi, G. Tóth, Activation of metrologically useful genuine multipartite entanglement, *New J. Phys.* **26** (2024), 023034.