On the Distance of Random Points from a Spherical Shell

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In this talk, we investigate the probability distribution of the distance between two independent random points selected from concentric spherical shells in $\mathbb{R}^d$. Employing the method of characteristic functions (Lord [2, 3]) as our primary analytical tool, we first derive the explicit density function for the distance between two independent, uniformly distributed random points in a d-dimensional spherical shell, using integrals involving Bessel functions. We also provide explicit, readily computable formulas for this density function in both the two- and three-dimensional cases. Additionally, as a demonstration of the method, we re-derive the density function of the distance for two beta-distributed random points in a d-dimensional unit ball; this particular distribution has been previously established via other methods. Finally, we extend our investigation to consider the density function of the distance between two independent random points following a beta distribution within concentric spherical shells, for specific values of the parameters β and d.

This is a joint work with Ferenc Fodor from the University of Szeged and it is based on the paper [1].

[1] Bakó–Szabó, A., Fodor, F., On the distribution of the distance of pairs of random points from a spherical shell, *Alea, Lat. Am. J. Probab. Math. Stat.* **23** (2026), 567–582.

[2] Lord, R. D., The distribution of distance in a hypersphere, *Ann. Math. Statistics* **25** (1954), 794–798.

[3] Lord, R. D., The use of the Hankel transform in statistics. I. General theory and examples, *Biometrika* **41** (1954), 44–55.