Monoidal Intervals for Idempotent Transformation Monoids

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Let A be a finite set and M be the submonoid of the monoid of all transformations on A. For $n \geq 1$, denote $\mathcal{O}_A^{(n)}$ the set of all n-ary operations on A. The set

$$\{f \in \mathcal{O}_A^{(n)} \colon f(m_1(x), \dots, m_n(x)) \in M \text{ for all } m_1, \dots, m_n \in M\}$$

is called the stabilizer of M. The lattice of all clones on a A is partitioned naturally into finitely many intervals whose minimal elements are clones generated by transformation monoids and whose maximal elements are the corresponding stabilizers. The problem of classifying transformation monoids according to the cardinalities of the corresponding monoidal intervals was posed by Á. Szendrei in [1].

We study a family of monoids for which all operations in the stabilizer can be represented by tuples of monotone Boolean operations. For 4-element sets, we have described all monoidal intervals in this family. Several questions remain open in the general case, but the symmetries of the family appear promising.

[1] Á. Szendrei, *Clones in universal algebra*, Séminare de mathématiques supérieures, Vol. 99, Les presses de l'Université de Montréal, Montréal, 1986.