Random Beta-Polytopes: Variance Bounds and Limit Theorems

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We consider the beta-distribution in $\mathbb{R}^d$, which is a rotationally symmetric probability distribution in the open unit ball. Special cases give back the uniform distribution in the unit ball or approach the uniform distribution on its boundary, the unit sphere.

The convex hull of n iid. beta-distributed random points is a random beta-polytope. We prove matching lower and upper bounds in order of magnitude on the variances of the intrinic volumes of random beta-polytopes as n goes to infinity. Using Stein's method, we prove central limit theorems and the variance upper bounds imply strong laws of large numbers.

We also prove variance lower and upper bounds of the same order of magnitude for the f-vector of a random beta-polytope, that is, the collection of the numbers of k-dimensional faces, $k = 0, \ldots, d-1$.

This is a joint work with Ferenc Fodor from the University of Szeged.

This research was supported by the University Research Scholarship Programme (EKÖP) no. EKÖP-452-SZTE, which has been implemented with the support provided by the Ministry of Culture and Innovation and the National Research, Development and Innovation Fund.

This research was also supported by NKFIH project no. 150151. Project no. 150151 has been implemented with the support provided by the Ministry of Culture and Innovation of Hungary from the National Research, Development and Innovation Fund, financed under the ADVANCED_24 funding scheme.