Construction of Rank-k Antipodal Sets

Andrey Kupavskii, **Akbar Shahkaramov**, Márton Naszódi
Eötvös Loránd University, Budapest, Hungary

A finite set $X \subset \mathbb{R}^d$ is called rank-k antipodal if for every subset $Y \subset X$ of cardinality $k+1$, the orthogonal projection of every point of $X \setminus Y$ onto the affine hull of Y lies in the convex hull of Y. Given the definition, we study the existence of large rank-k antipodal sets in high-dimensional Euclidean spaces. Using a probabilistic method, we consider independent, uniformly distributed points on the unit sphere $\mathbb{S}^{d-1}$, and check for conditions under which the resulting configuration is rank-k antipodal. We prove that, for every fixed k, there exists rank-k antipodal set in $\mathbb{R}^d$ whose cardinality grows exponentially with the dimension. More precisely, we obtain an explicit lower bound of the form $e^{\Omega(d/k^4)}$ for the maximum cardinality of rank-k antipodal set. The proof combines the geometric properties of random simplices with probabilistic estimates for random points on the sphere.