On the Diophantine Equation $u_n = u_l^k(Au_r^m + B)$

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Let $\{u_n\}_{n\geq 0}$ be a Lucas sequence having a companion polynomial $f(x) = x^2 - ax - b$ with $b = \pm 1$. For given integers A, B with $A \neq 0$, we consider the Diophantine equation

$$u_n = u_l^k(Au_r^m + B) \tag{1}$$

in non-negative integer unknowns (n, l, k, r, m).

The aim of this talk is twofold. On the one hand, we present a general effective finiteness theorem concerning equation (1). More precisely, we prove that there exists an effectively computable constant $C = C(\{u_n\}_{n\geq 0}, A, B)$ depending only on the sequence $\{u_n\}_{n\geq 0}$ and the parameters A, B such that $\max\{n, l, k, r, m\} < C$. On the other hand, by combining this general effective finiteness result with some reduction methods (e.g. LLL algorithm) we determine all solutions of equation (1) when $\{u_n\}_{n\geq 0} = \{F_n\}_{n\geq 0}$ is the Fibonacci sequence and $1 \leq A \leq 10, |B| \leq 5$. This way we give a broad extension of a recent result of Ibrahimov and Mahmudov [1], who solved a special case of equation (1) which was originally proposed by Florian Luca.

We study the Diophantine equation

$$u_n = u_l^k(Au_r^m + B),$$

where $\{u_n\}_{n\geq 0}$ is a Lucas sequence with $b = \pm 1$. We prove an effective finiteness result giving an explicit upper bound for all variables. Moreover, we completely solve the equation in the case of the Fibonacci sequence for $1 \leq A \leq 10, |B| \leq 5$, using reduction methods.

[1] S. S. Ibrahimov, N. I. Mahmudov, On the equation $F_n = F_l^k(F_l^m - 1)$, *Publ. Math. Debrecen,* (to appear).