Objects of Hodge Theory

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The aim of this talk is to introduce the basic objects and results of Hodge theory, starting from the mid-1900s. One of the earliest results was obtained by Narasimhan and Seshadri in 1965, who proved in [2] that stable vector bundles on a compact Riemann surface are in one-to-one correspondence with irreducible Hermitian connections of constant central curvature on the given bundle. In this talk, I define the notions of stability for vector bundles, Hermitian connections, and the moduli spaces of vector bundles and connections.

The theory was extended in another direction by Hitchin in 1987, when he introduced the notion of a Higgs bundle in [1], which generalizes the concept of a holomorphic vector bundle. Moreover, it turned out that the stability of a Higgs bundle is linked to the existence of solutions to the so-called Hitchin's equations. These are physically motivated nonlinear elliptic partial differential equations for a connection and a tensor field (called the Higgs field) on a vector bundle over a complex curve. I give the definitions of the relevant objects and state the theorem describing the homeomorphism between the moduli spaces of stable Higgs bundles and irreducible flat connections. This is the theorem of Hitchin, Simpson, Donaldson, and Corlette, also known as the non-abelian Hodge correspondence.

At the end of the talk, I discuss directions of this theory that I studied during my PhD research.

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[1] N. J. Hitchin, The self duality equations on a Riemann surface, *Proc. London Math. Soc. (3)*, **55** (1987), 58–126.

[2] M. S. Narasimhan, C. S. Seshadri, Stable and unitary vector bundles on a com- pact Riemann surface, *Ann. of Math.*, **82** (1965), 540–564.