Shannon Capacity and Graph Operations

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The Shannon capacity of graphs is a notoriously difficult parameter to compute. Its exact value is unknown even for some small graphs, such as the cycle of length 7. The study of graph parameters that provide upper bounds on the Shannon capacity remains an important part of zero-error information theory. This talk presents an investigation of the relationship between the Mycielski construction and certain well-behaved upper bounds on the Shannon capacity, namely parameters belonging to the asymptotic spectrum of graphs.

The Mycielski construction was one of the first examples showing that the clique number $\omega(G)$ and the chromatic number $\chi(G)$ of a graph can be arbitrarily far apart. Namely, if $M(G)$ denotes the Mycielski construction of a graph G, then $\omega(M(G)) = \omega(G)$ and $\chi(M(G)) = \chi(G) + 1$.

This naturally raises the question of how the construction affects other graph parameters that serve as upper bounds on the Shannon capacity.

For one of the best-known such parameters, the fractional chromatic number, it was shown in [3] that the Mycielski construction behaves in a particularly regular way: $\chi_f(M(G)) = \chi_f(G) + \frac{1}{\chi_f(G)}$. Later, a more general formula was proved for $M_r(G)$ [4], the generalized Mycielski construction:

$$\chi_f(M_r(G)) = \chi_f(G) + \frac{1}{\sum_{k=0}^{r-1}(\chi_f(G) - 1)^k}.$$

Following this line of research, we showed in [1] that a similar formula can be given for another important upper bound, the theta number $\bar{\vartheta}(M(G))$, and we also obtain analogous results for the fractional Haemers bound $\mathcal{H}_f(G; \mathbb{F})$, depending on the graph G and the field $\mathbb{F}$ [2].

The talk will focus on these formulae and on how they illustrate the behaviour of Shannon-capacity upper bounds under graph operations.

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