

CONVERGENCE RATE ANALYSIS OF RATIO CONSENSUS ALGORITHMS WITH COLUMN-ALLOWABLE MATRICES

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In distributed computing one fundamental subroutine is distributed averaging of initial values along a network using local communication. It can be a building block for example to calculate linear synopses, random sampling and quantile computation. Real world networks can have various imperfections which were addressed in previous works step-by-step. For handling asynchronous, directed communication - when doubly stochasticity cannot be assumed for the underlying matrix process - the protocol *push-sum* was introduced by [1] and later extended by [2] as *weighted gossip*. Later it was referred as *ratio consensus*. Most theory regarding ratio consensus discusses convergence without exact rates.

Gerencsér and Gerencsér in [3] proved that the rate of convergence was connected to the difference of the top two Lyapunov exponents of the process of transition matrices. However, their theorems need the transition matrices to be *allowable* (at every step there are both in- and outgoing communication) and the process of transition matrices being *sequentially primitive*: after an almost surely finite stopping time each node contains a portion of the original value of every other.

To handle other challenges with the communication, variants of the push-sum protocol were introduced. Hadjicostis et al. in [4] considered the cases when there can be packet losses which are not directly detected by neither the sender nor the receiver, and the one when the knowledge of outgoing edges is imprecise. However, for their solution the condition of allowability of the transition matrices and sequential primitivity of the matrix process do not hold anymore, thus the results on the exact rate of convergence cannot be used.

Motivated by the example above, in this work we extend the results of Gerencsér and Gerencsér [3] to be able to handle the push-sum protocol when the allowability and sequential primitivity condition is weakened.

The research was supported by NRD (National Research, Development and Innovation Office) grant KKP 137490.

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