Rubik's Cube Variants, Graphs and Groups

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While the classic Rubik's Cube (and a set of its highly symmetric variants) can be described with groups, most twisty puzzles are too irregular in their nature to be interpreted as such. This is a formal attempt to build a rigorous framework for analyzing the state spaces of the widest possible variety of twisty puzzles through graph theory. By establishing fundamental conventions rooted in physical observation, we demonstrate that the state space of any twisty puzzle can be fully described as a graph, where nodes represent states and labeled edges denote singular transitions. Central to our approach is the classification of the states using graph automorphisms, which allow for the isolation of a specific subset of states that share combinatorial symmetry with the puzzle's solved state. We prove that this subset exhibits rigorous algebraic properties, namely that these configurations form a group under a set of move sequences. This, combined with the local structure of the graph, makes for a practical tool for describing a much larger set of puzzles than before. Finally, this work outlines future objectives aimed at connecting geometric properties to graph structures, and determining the precise mathematical conditions under which these state spaces can be characterized as Cayley graphs with missing edges.