Heavy-Tailed Critical Galton–Watson Processes with Immigration

Péter Kevei, **Kata Kubatovics**
University of Szeged, Szeged, Hungary

We study critical Galton–Watson branching process with immigration in the heavy tailed setting. Specifically, we assume that the offspring distribution belongs to the domain of attraction of a $(1+\alpha)$-stable law with $\alpha \in (0,1)$, while the immigration distribution satisfies one of the following two conditions: either it has finite mean, or it belongs to the domain of attraction of a β-stable law with $\beta \in (\alpha, 1)$. Under these assumptions, we prove that the stationary distribution of the process has a regularly varying tail. We then carry out a detailed analysis of the stationary process itself. In particular, we identify its tail process and prove a stable central limit theorem for the partial sums. A noteworthy feature of our results is that the norming sequence is different from the one corresponding to the tail of the stationary law. In particular, the extremal index of the process is 0.

The talk is based on joint work with Péter Kevei [1].

This research was supported by the University Research Scholarship Program of the Ministry of Culture and Innovation, funded by the National Research, Development and Innovation Fund.

[1] P. Kevei, K. Kubatovics, Heavy-tailed critical Galton–Watson processes with immigration, *https://arxiv.org/abs/2510.02004*