

# TREE REPRESENTATIONS OF TAME QUIVERS

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We present a fundamental problem of representation theory of path algebras over tame quivers, namely to provide a complete classification of exceptional modules (indecomposable and without self-extensions) in terms of explicit linear spaces and maps.

We give such a classification (see [1] and [2]) in the case of tame quivers  $\tilde{\mathbb{E}}_6$  and  $\tilde{\mathbb{D}}_m$  with canonical orientation, for an arbitrary  $m > 3$ . Matrix forms for  $\tilde{\mathbb{D}}_m$  were obtained by reduction to  $\tilde{\mathbb{D}}_6$ .

All of the given representations are tree representations, which are defined by having only the elements 0 and 1 such that the number of ones is exactly  $d - 1$  for a module of length  $d$ , resulting in the simplest possible description. Moreover, they are field independent, thus giving an affirmative answer to a question raised by Ringel in [3] regarding the existence of such representations.

The correctness of these findings, detailed in [5], was confirmed through an exhaustive proof (spanning several thousand pages), mostly done by a purposefully developed computer proof assistant.

In addition (see [4]), we verify computationally a conjecture on the field independence of tree representations of tame quivers  $\tilde{\mathbb{D}}_4$ ,  $\tilde{\mathbb{D}}_5$ , and  $\tilde{\mathbb{E}}_6$ , which includes a large class of exceptional representations, in particular all the regular non-homogeneous ones.

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