Asymptotics of Random Tilings of the Aztec Diamond

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I will give an introductory overview of domino tilings of the Aztec diamond, and of key ideas behind their asymptotic analysis. These tilings have several equivalent interpretations, such as perfect matchings of a bipartite graph or configurations of diatomic molecules (dimers) on a planar lattice, and they are closely related to the square ice model from statistical physics. Various aspects of these tilings have been studied through techniques from enumerative combinatorics, relying on their connections to plane partitions, alternating sign matrices and monotone triangles [1, 3], while others can be understood via height functions [1], families of non-intersecting lattice paths, and point processes [5]. The random generation of tilings according to a given distribution also plays a crucial role. For the uniform distribution, this can be achieved by the shuffling algorithm [1, 2], with generalizations available for more intricate distributions [4].

A tiling consists of four types of dominoes, and has five clearly distinguishable regions. In the four ordered (or frozen) regions only dominoes of one type appear, while in the temperate region all four types appear in a disordered manner. If the size of the Aztec diamond goes to infinity, the boundary of the temperate region is close to the inscribed circle. This is the so-called Arctic Circle phenomenon. The growth under the shuffling algorithm of, for example, the upper frozen region (or north polar region) is equivalent to the dynamics of the TASEP model [2].

These ideas are part of the broader framework underlying my current research, and I will present them with an emphasis on conceptual understanding rather than technical details, aimed at a general mathematical audience.

[1] Elkies, N., Kuperberg, G., Larsen, M. et al., Alternating-Sign Matrices and Domino Tilings (Part I), *Journal of Algebraic Combinatorics* **1** (1993), 111–132.

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[3] Bressoud, D. and Propp, J., How the Alternating Sign Matrix Conjecture Was Solved, *Notices of the AMS.* **46(6)** (1999), 637–646.

[4] Propp, J., Generalized Domino-Shuffling, *Theoretical Computer Science* **303** (2003), 267–301.

[5] Johansson, K., The Arctic Circle Boundary and the Airy Process, *The Annals of Probability* **33(1)** (2005), 1–30.