

GROUP RINGS WITH METABELIAN UNIT GROUPS IN CHARACTERISTIC 2

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Let F be a field of characteristic $p > 0$ and let G be a non-abelian group. Denote by $U(FG)$ the unit group of the group ring FG . In 1991, Shalev [6] provided a necessary and sufficient condition for $U(FG)$ to be metabelian (i.e., for the commutator subgroup of $U(FG)$ to be abelian) in the case where G is finite and $p > 2$. A few years later, Kurdics [4], as well as Coleman and Sandling [2] independently, extended this result to the case $p = 2$. Namely, they proved that when $p = 2$ and G is finite, $U(FG)$ is metabelian if and only if either the commutator subgroup G' of G is a central elementary abelian 2-group of order at most 4, or F is the field of 2 elements and G belongs to a certain class of non-nilpotent groups.

For odd characteristic, the finiteness assumption on the order of G has already been relaxed: Catino and Spinelli [1] showed that Shalev's theorem remains valid when G is a torsion group, and in 2022, Juhász and Spinelli [3] investigated the case where G contains elements of infinite order, identifying additional cases.

In characteristic 2, Catino–Spinelli [1] and Mozgovoj [5] proved that the Kurdics, Coleman–Sandling theorem also holds for torsion groups. Moreover, Mozgovoj demonstrated that the result remains valid even when G contains elements of infinite order, provided that G is non-nilpotent. The aim of this presentation is to continue these investigations in the nilpotent case. The main result provides a complete characterization of groups G for which $U(FG)$ is metabelian.

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