Matrix Tuples with Linearly Dependent Invariant Subspaces

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For a given partition $k_1, \ldots, k_l$ of n, we proved that the set of l-tuples of $n \times n$ complex matrices $(A_1 \ldots, A_l)$ such that there exist subspaces $V_1, \ldots, V_l$ with V_i invariant under A_i, $\dim V_i = k_i$ and $\langle V_1, \ldots, V_l \rangle \neq \mathbb{C}^n$ is an algebraic hypersurface.

I will talk about the proof of this statement and the defining polynomial of this hypersurface [2] starting with some special cases [1], for example:

Theorem 1. *For $n \times n$ matrices A and B, the following are equuivalent:*

1. *A has a 1 dimensional invariant subspace contained in some $n-1$ dimensional invariant subspace of B*

2. $$\det \begin{pmatrix} I & B & \cdots & B^{n-1} \\ A & BA & \cdots & B^{n-1}A \\ \vdots & \vdots & & \vdots \\ A^{n-1} & BA^{n-1} & \cdots & B^{n-1}A^{n-1} \end{pmatrix} = 0$$

[1] Tamás Bencze, Péter E. Frenkel, The triangulant, *Linear Algebra and its Applications* **709** (2025), 92–110.

[2] Tamás Bencze, Matrix tuples with linearly dependent invariant subspaces, *arXiv* 2604.22733 (2026)