Generalized Poly-Bernoulli Numbers

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Bernoulli numbers are well-known in mathematics. Their generalization, the poly-Bernoulli numbers were defined in the 1990s by a Japanese mathematician, Kaneko. If the new parameter is nonpositive, then poly-Bernoulli numbers take positive integer values. They can be expressed in terms of Stirling numbers of the second kind in two different ways. In the next decades, several authors found combinatorial interpretations of poly-Bernoulli numbers. We give a brief overview of these results in the beginning of our talk.

Stirling numbers have a widely studied variant, the so-called r-Stirling numbers. Our goal was to find a generalization of poly-Bernoulli numbers which can be expressed by r-Stirling numbers of the second kind and has mathematically relevant combinatorial meanings. We introduce r-poly-Bernoulli numbers and derive their exponential generating function. We present three interpretations of them:

- They count certain permutations with some restrictions, the generalized Vesztergombi permutations.
- They also count directed acyclic orientations of certain complete multipartite graphs.
- Finally, they count certain families of lonesum matrices arising from discrete tomography.

This is a joint work with Gábor Nyul.