Removable Sets for Fractional Heat and Fractional Bessel-Heat Equations

Mouna Chegaar
Budapest University of Technology and Economics, Budapest, Hungary

This talk concerns removable sets for fractional heat-type equations in the L^p setting. The main question is whether a compact set K may support a nontrivial singularity of a solution, or whether every solution defined outside K necessarily extends across K as a distributional solution of the same equation. Since Lebesgue measure is not sufficiently fine to describe such exceptional sets, the problem is naturally formulated in terms of capacities adapted to the corresponding parabolic operator.

We consider the fractional parabolic operator

$$L_{\gamma,a} = (-\Delta_a)^{\gamma/2} + \partial_t,$$

where Δ_a denotes either the classical Laplacian or the Bessel–Laplace operator on the positive orthant. In the classical case, the analysis relies on Fourier methods and ordinary convolution. In the Bessel setting, these tools are replaced by the Hankel transform, Bessel translation, and Bessel convolution. This operator-adapted framework allows one to construct the corresponding fractional heat kernels and parabolic capacities.

The aim of the talk is to present L^p-removability criteria for compact sets for both fractional heat equations and fractional Bessel–heat equations. The main results show that removability is characterized by the vanishing of suitable capacities. Thus, the singular sets of these fractional parabolic equations are described through a potential-theoretic condition that reflects the geometry of the underlying operator.

This work is joint with Á. P. Horváth.