Certification of Absolutely Entangled Sets in Prepare-and-Measure Scenarios

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Entanglement is a central resource in quantum information science, underpinning key advantages in communication, computation, and cryptography. However the notion of entanglement is always defined among subsystems and it depend on the choice of global reference frame. This motivates the study of absolutely entangled sets (AES), defined as collections of quantum states for which, under any global unitary transformation, at least one state remains entangled [1].

In this work, we develop a semi-device-independent framework for certifying absolutely entangled sets (AES) in a prepare-and-measure communication scenario. Our approach relies solely on an assumption of the underlying Hilbert-space dimension, without requiring any knowledge of the internal functioning of the preparation and measurement devices. Within this framework, we construct correlation witnesses whose violation certifies the presence of AES, while all separable preparations are guaranteed to satisfy the corresponding bounds. To obtain tight separable bounds, we combine alternating convex optimization, yielding lower bounds, with a semidefinite-programming relaxation hierarchy that provides upper bounds. We extend moment-matrix based prepare-and-measure techniques of the Navascués–Vértesi type [2] by introducing a modified sampling strategy that enforces separable structure at the level of the operator basis.

In particular, we have constructed a random access code (RAC) witness tailored to AES detection. We analyze a one-parameter family of quantum states interpolating between entangled and separable regimes and show that violation of the separable bound occurs within a finite parameter range, thereby certifying AES. In addition, we formulate a general procedure for constructing AES witnesses and certifying arbitrary target AES in prepare-and-measure settings. We further illustrate the applicability of the proposed framework through several representative examples of target AES. Taken together, our results establish a systematic and experimentally feasible approach for the certification of arbitrary AES in semi-device-independent quantum information protocols.

[1] Y. Cai, B. Yu, P. Jayachandran, N. Brunner, and V. Scarani, and JD Bancal, Entanglement for any definition of two subsystems, *Physical Review A* **103** (2021).

[2] M. Navascués, and T. Vértesi, Bounding the Set of Finite Dimensional Quantum Correlations, *Physical Review Letters* **115** (2015).