Certifying Asymmetry in the Configuration of Finite Sets of Qubits

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Building on our asymmetry-certification protocol for configurations of three qubit states [1], we extend the approach to arbitrary finite sets of qubit states whose Bloch vectors form a three-dimensional configuration. Our construction starts from the self-testing witness of Drótos et al., a linear functional whose maximal quantum value self-tests a target set of qubit preparations [2]. We then optimize the same witness over all mirror-symmetric configurations, obtaining a mirror-symmetry bound. Any violation of this bound certifies that the observed set of preparations cannot admit a mirror-symmetry plane, and therefore that the target configuration is asymmetric. The computed mirror-symmetry bound is obtained by relaxing the original nonconvex optimization problem to a conic convex program, and therefore gives an upper bound on the value attainable by mirror-symmetric configurations.

Based on numerical evidence, we conjecture that these mirror-symmetry bounds are tight and that the corresponding optima can be attained by pure qubit preparations, since in all cases studied the optima of the relaxed conic program are feasible for the original nonconvex problem. We also show how upper bounds for configurations with multiple mirror planes can be derived from the corresponding single-plane mirror-symmetry bounds.

For a given set of N qubit states, the gap $\Delta = Q_{\max} - Q_{\mathrm{mirror}}$ between the quantum maximum, which self-tests the target set of qubit preparations, and the mirror-symmetry bound provides an operational quantifier of asymmetry. We use a heuristic search to identify configurations with large asymmetry gaps, which are expected to be the most robust candidates for certification on noisy quantum devices. In this way, we compute maximal asymmetry gaps for several numbers of qubit states.

Finally, we experimentally certify asymmetry for several sets of pure qubit states on publicly accessible quantum processors.

[1] A. Taoutioui, G. Drótos, T. Vértesi, Certifying asymmetry in the configuration of three qubits, *New J. Phys.* **27** (2025), 124501.

[2] G. Drótos, K. F. Pál, A. Taoutioui, T. Vértesi, Towards minimal self-testing of qubit states and measurements in prepare-and-measure scenarios, *New J. Phys.* **26** (2024), 063012.