Threshold Dynamics in a Time-Delayed Logistic Model of Cell Populations

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We extend the delayed logistic cell-population model of Baker and Röst [1], building on the general theory of delay differential equations [2, 3]. The generalized equation incorporates distributed delays expressed via both discrete and integral terms, and explicitly features the death rates of dividing and motile cells as parameters.

We first establish well-posedness, along with the nonnegativity and boundedness of biologically relevant solutions. We then derive an explicit threshold parameter that determines the stability of the zero equilibrium and the existence of a positive equilibrium. When the zero equilibrium is stable, no positive equilibrium exists and the cell population goes extinct. When the zero equilibrium is unstable, there exists exactly one positive equilibrium, which is stable; in this system we prove uniform strong persistence of the population.

Our results quantify how the death rates of dividing and motile cells, as well as the delay representing the duration of the division process, shape the system's global dynamics.

[1] R. E. Baker, G. Röst, Global dynamics of a novel delayed logistic equation arising from cell biology, *J. Nonlinear Sci.* **30** (2020), 397–418.

[2] H. Smith, *An introduction to delay differential equations with applications to the life sciences*, Texts in Applied Mathematics, Vol. 57, Springer, New York, 2010.

[3] Y. Kuang, *Delay differential equations: with applications in population dynamics*, Mathematics in Science and Engineering, Vol. 191, Academic Press, San Diego, CA, 1993.