Cofinal Types of Topological Groups

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We will present some basic results about cofinal types of topological groups. When we say the cofinal type of a topological group, we have in mind the cofinal type of a local base of the identity in a topological group. Note that every two local bases of the identity are cofinally similar so this definition makes sense. In this setup the well-known Birkhoff-Kakutani Theorem states that a topological group is metrizable if and only if it is Tukey reducible to a countable directed set.

We study which properties are preserved by Tukey order (for example being Fréchet, countable tightness, being a P-group). We also extend a result from [2] by proving that each locally countably compact, strongly basically generated, and countably tight topological group is metrizable. We also show a general result which gives a sufficient condition for a group of tightness κ to be Tukey reducible to a cardinal λ. As a corollary, we see that under the P-ideal dichotomy, if $\mathfrak{p} > \omega_1$, then every countably tight topological group of weight at most ω_1 is either metrizable or $[\omega_1]^{<\omega}$ is Tukey reducible to it.

This is a joint work with Boriša Kuzeljević, University of Novi Sad, Serbia.

[1] B. Kuzeljević, S. Milošević, Cofinal Types of Topological Groups, *ArXiv* (April 2024)

[2] Alan Dow and Ziqin Feng, Compact spaces with a P-base, *Indag. Math. (N.S.)* **32(4)** (2021), 777–791.