Tree Representations of Tame Quivers $\widetilde{\mathbb{D}}_m$ and $\widetilde{\mathbb{E}}_6$

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This work is devoted to the important problem of representation theory of path algebras over tame quivers, namely to provide a comprehensive classification of exceptional (indecomposable and without self-extensions) modules in terms of concrete linear spaces and linear maps.

We give such a classification (see [1] and [2]) in the case of tame quivers $\widetilde{\mathbb{E}}_6$ and $\widetilde{\mathbb{D}}_m$ with canonical orientation, for an arbitrary $m > 3$. In the case of $\widetilde{\mathbb{D}}_m$ we obtain the desired matrix forms by reducing the problem to the $\widetilde{\mathbb{D}}_6$ case.

All of the given representations are so-called tree representations, i.e. they only have the elements 0 and 1, such that the number of ones is exactly $d-1$, where d is the length of the module, which results in the simplest possible description of indecomposable exceptional modules. Moreover, they are field independent, thus giving an affirmative answer to a question raised by Ringel in [3], where he proves the existence of tree representations for exceptional modules.

Furthermore (see [4]) we verify computationally a conjecture on the field independence of tree representations of tame quivers $\widetilde{\mathbb{D}}_4$, $\widetilde{\mathbb{D}}_5$ and $\widetilde{\mathbb{E}}_6$, with dimension vector bounded by the minimal radical vector of the quiver. This includes a large class of exceptional representations, in particular all the regular non-homogeneous exceptionals. In addition we also present some thought-provoking findings, which further confirm the combinatorial nature of the category of representations of tame quivers.

The verification of the correctness of our results (see [5]) consists of many cases and steps (totalling thousands of pages) and it is mostly done by a purposefully developed computer proof assistant.

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[1] Sz. Lénárt, Á. Lőrinczi, Cs. Szántó and I. Szöllősi, Tree representations of the quiver $\widetilde{\mathbb{D}}_m$, 0*Colloq. Math.*, **167** (2022), 261—302.

[2] Sz. Lénárt, Á. Lőrinczi and I. Szöllősi, Tree representations of the quiver $\widetilde{\mathbb{E}}_6$, *Colloq. Math.*, **164** (2021), 221–250.

[3] C. M. Ringel, Exceptional modules are tree modules, *Linear Algebra Appl.*, **275-276** (1998), 471–493.

[4] Á. Lőrinczi, *On the combinatorial nature of tree representations of Euclidean quivers*, Mathematica, **65 (88)** (2023), 1, 85–93.

[5] Sz. Lénárt, Á. Lőrinczi, Cs. Szántó and I. Szöllősi, Proof of the tree module property for exceptional representations of tame quivers, *arXiv*, 2001.00016v3, 2021