An Efficient algorithm to Compute the Toughness in Graphs with Bounded Treewidth

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Let t be a positive real number. A graph is called *t-tough* if the removal of any vertex set S that disconnects the graph leaves at most $|S|/t$ components. The toughness of a graph is the largest t for which the graph is t-tough. We prove that toughness is fixed-parameter tractable parameterized with the treewidth. More precisely, we give an algorithm to compute the toughness of a graph G with running time $\mathcal{O}(|V(G)|^3 \cdot \mathrm{tw}(G)^{2\mathrm{tw}(G)})$ where $\mathrm{tw}(G)$ is the treewidth. If the treewidth is bounded by a constant, then this is a polynomial algorithm. There are quite a few results on the computational aspects of toughness.

In [2] Bauer et al. proved that, For any positive rational number t, the problem t-Tough is coNP-complete. On the other hand, for some graph classes, we do have polynomial algorithms. For example the class of claw-free graphs [4], class of split graphs [5] and the class of $2K_2$-free graphs [6].

There are many decision problems on graphs that are NP-complete in general but can be solved in polynomial time for graphs with bounded treewidth using dynamic programming [7].

Computing the treewidth of an arbitrary graph is an NP-hard problem [1]. However, if the treewidth is bounded, it can be computed in polynomial time [3].

We use a dynamic programming approach to solve this modified problem, assuming that very nice tree decomposition of G is also given.

[1] S. Arnborg, D.G. Corneil, A. Proskurowski, Complexity of finding embeddings in a k-tree, *SIAM Journal on Algebraic Discrete Methods* **8** (2):277–284, (1987).

[2] D. Bauer, S.L. Hakimi, E. Schmeichel, Recognizing tough graphs is NP-hard, *Discrete Applied Mathematics* **28** 191–195, (1990).

[3] H.L. Bodlaender, A linear time algorithm for finding tree-decompositions of small treewidth, *SIAM Journal on Computing* **25** (6):1305–1317, (1996).

[4] M.M. Matthews, D.P. Sumner, Hamiltonian results in K1,3-free graphs, *Journal of Graph Theory* **8** (1):139–146 (1984).

[5] G.J. Woeginger,The toughness of split graphs, *Discrete Mathematics* **190** (1–3):295– 297 (1998).

[6] H.J. Broersma, V. Patel, A. Pyatkin,On toughness and hamiltonicity of 2K2-free graphs, *Journal of Graph Theory* **75** (3):244–255 (2014).

[7] M. Cygan, F.V. Fomin, L. Kowalik, D. Lokshtanov, D. Marx, M. Pilipczuk, M. Pilipczuk, S. Saurabh, Parameterized Algorithms, *Springer* ISBN 978-3-319-21274-6 (2015).