On the Variance of Generalized Random Polygons

Balázs Grünfelder
University of Szeged, Szeged, Hungary

We prove asymptotic lower and upper bounds on the variance of some geometric quantities of generalized random polygons in various probability models in and around a smooth convex disc. The common element in these models is that the generalized random polygons are formed by the intersection of all translates of a suitable fixed convex disc containing a sample of i.i.d. random points.

In the first model the generalized random polygons are the so-called disc-polygons, i.e. the intersection of circular discs of radius r. We prove asymptotic upper bounds on the variance of the number of vertices and missed area of inscribed random disc-polygons in smooth convex discs whose boundary is C^2_+. The established lower bounds are of the same order as the upper bounds proved previously in [1].

In the second model we consider the L-polygons, i.e. the intersection of all translates of another suitable fixed convex disc L that contain the sample. We prove upper bounds on the variance of the number of vertices and missed area of random L-polygons under different conditions on the curvatures. We also transfer some of our results to a circumscribed variant of this model.

Upper bounds on variances lead to strong laws of large numbers, lower bounds may be used to prove central limit theorems.

The talk is based on joint works with F. Fodor (Szeged) and V. Vígh (Szeged).

This research was supported by the ÚNKP–23–3–New National Excellence Program of the Ministry for Culture and Innovation from the source of the National Research, Development and Innovation Fund.

This research was also supported by project TKP2021-NVA-09. Project no. TKP2021-NVA-09 has been implemented with support provided by the Ministry of Culture and Innovation of Hungary from the National Research, Development and Innovation Fund, financed under the TKP2021-NVA funding scheme.

[1] F. Fodor, V. Vígh, Variance estimates for random disc-polygons in smooth convex discs, *J. Appl. Probab.* **55** (2018), 1143–1157.