First Order Systems of Ordinary Differential Equations Allowing a Given 3-Dimensional Lie Group as a Subgroup of Their Symmetry Group

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Many research are done to find Lie symmetries for existing systems in physical and engineering sciences ([4]). Applications of symmetry groups to understand and find the solutions of differential equations go back to work of Sophus Lie ([5], [7]). He classified the Lie groups given by the Lie algebras of their infinitesimal generators which act either on the complex line or on the complex plane (cf. [6], pp. 767–773, [3], p. 1164). Furthermore, he improved a method in [6], Section X, pp. 243–248, to receive the ordinary differential equations which admit a given Lie group as a group of their symmetries. Using the method of Lie, we formulate the appropriate necessary condition for a system of first order ordinary differential equations allowing a given Lie group as a group of their symmetries.

In this talk I would like to show how we determine the first order systems of ordinary differential equations such that their group of symmetries contains a three-dimensional Lie subgroup G. We represent the basis vectors of the Lie algebra $\mathfrak{g}$ of G by vector fields in the three-dimensional real space ([6], [2]). Two cases are distinguished according whether the infinitesimal generators of $\mathfrak{g}$ do not contain any component or contain component with respect to the independent variable of the system.

Our results can be found in [1].

[1] K. Ficzere, Á. Figula, First order systems of ordinary differential equations allowing a given 3-dimensional Lie group as a subgroup of their symmetry group, submitted in (2024), pp. 22.

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[3] A. González-López, N. Kamran, P. J. Olver, Lie Algebras of Differential Operators in Two Complex Variables, *Amer. J. Math.* **114** (1992), 1163–1185.

[4] N. H. Ibragimov (ed.), *CRC Handbook of Lie Group Analysis of Differential Equations*, Applications in Engineering and Physical Sciences, Vol. 2, CRC Press, Boca Raton, 1995.

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