Some Geometric Aspects of Non-Abelian Hodge Theory

Miklós Eper, Szilárd Szabó (supervisor)
University of Technology and Economics, Budapest, Hungary

The main objects of our research are the moduli space of Higgs bundles on $\mathbb{C}P^1$ with three logarithmic points (called the Dolbeault moduli space) and the moduli space of the representations of the fundamental group of the three-punctured sphere (called the Betti moduli space). We investigate these spaces with specially settled parameters, with which they are known to be complex varieties of dimension 4. The study is related to Hitchin's WKB problem [1], which roughly reads as follows: consider a $\mathbb{C}^\times$ orbit in the Hitchin base, and its lift to the Dolbeault space. Consider the family of Higgs bundles corresponding to this lift, and determine the behaviour of the associated transport matrices of the family of representations in the Betti space, as converging to the infinity along the given $\mathbb{C}^\times$ orbit.

A more specific topic in the theory of these spaces is related to the so-called Geometric P=W conjecture [1]. It asserts the existence of a homotopy commutative diagram, including the non-abelian Hodge correspondence, the Riemann Hilbert correspondence between the Dolbeault and the Betti moduli spaces, the Hitchin fibration over the Hitchin base and Simpson's natural map from the compactification of the Betti space to the body of the nerve complex of its compactifying divisor.

We show, that the Geometric P=W conjecture holds for rank 3 parabolic Higgs bundles and SL(3,$\mathbb{C}$)-character variety over the three-punctured sphere, and provide a self-contained proof for the homotopy type of the above mentioned nerve complex being S^1. Therefore we describe the SL(3,$\mathbb{C}$) character variety of the three-punctured sphere in general, and provide a special analysis of the character variety with trace coordinates, which were introduced for SL(3,$\mathbb{C}$) character varieties by S. Lawton [2]. Our main technique is T. Mochizuki's asymptotic abelianization method for some large-scale analysis of the Dolbeault space [3].

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