Asymptotic Behavior of Some Critical Decomposable Multi-type Galton–Watson Processes with Immigration

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We study the asymptotic behavior of some critical 2- and 3-type Galton–Watson processes with immigration when the process's offspring mean matrix is reducible, in other words, when the process is decomposable. Under second or fourth order moment assumptions on the offspring and immigration distributions, we prove that a sequence of appropriately scaled random step processes formed from a critical decomposable 2-type Galton–Watson process with immigration converges weakly, see Barczy et al. [1]. The limit process can be described using independent squared Bessel processes, their integral processes, and possibly the unique stationary distribution of an appropriate subcritical single-type Galton–Watson process with immigration. We also prove a similar result for a critical decomposable 3-type Galton–Watson process with immigration whose offspring mean matrix is lower triangular with all the diagonal entries 1. In this case, the limit process can be described using independent squared Bessel processes, the linear combinations of their integral processes, and possibly their 2-fold iterated integral processes. In the proofs we use limit theorems for martingale differences towards a diffusion process due to Ispány and Pap [2].

This is a joint work with Mátyás Barczy from HUN-REN–SZTE Analysis and Applications Research Group, Bolyai Institute, University of Szeged, Szeged, Hungary.

Mátyás Barczy is supported by the Ministry of Culture and Innovation of Hungary from the National Research, Development and Innovation Fund, project no. TKP2021-NVA-09.

- M. BARCZY, D. BEZDÁNY, G. PAP, Asymptotic behaviour of critical decomposable 2-type Galton-Watson processes with immigration. *Stochastic Processes and their Applications* 160 (2023) 318–350.
- [2] M. ISPÁNY, G. PAP, A note on weak convergence of random step processes, Acta Mathematica Hungarica, 126(4) (2010), 381–395.