

# NUMBERS OF A HYPERGRAPH AND ITS 1-INTERSECTION GRAPH

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A well known problem from an excellent book of Lovász [1] states that any hypergraph with the property that no pair of hyperedges intersect in exactly one vertex can be properly 2-colored. Motivated by this as well as recent works of Keszegh [2] and of Gyárfás et al [3] we study the 1-intersection graph of a hypergraph. The 1-intersection graph encodes those pairs of hyperedges in a hypergraph that intersect in exactly one vertex. More precisely, the vertex set of the 1-intersection graph is the set of hyperedges of the hypergraph. Two hyperedges are adjacent if and only if their intersection has exactly one element.

Keszegh recently investigated the colorings of directed hypergraphs, meanwhile the main result of the paper of Gyárfás et al. is the following: for 3-uniform hypergraphs the chromatic number is always at most as large as the chromatic number of their 1-intersection graph, if the 1-intersection graph is nonempty. We try to generalise this result to arbitrary hypergraphs by omitting the uniformity constraint. We prove for  $k \in \{2, 4\}$  that all hypergraphs whose 1-intersection graph is  $k$ -partite can be properly  $k$ -colored and show counterexamples for any odd  $k$  values.

This is a joint work with Nathan W. Lemons from the Los Alamos National Laboratory.

This research was supported by the ÚNKP-23-4-SZTE-628 New National Excellence Program of the Ministry for Culture and Innovation from the source of the National Research, Development and Innovation Fund, and also by the OTKA grant no. SNN 132625.

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