Analytic Verification of Stable Periodic Orbits: a Study on Mackey-Glass Type Equations

Gábor Benedek
Bolyai Institute, University of Szeged, Szeged, Hungary

We study Mackey-Glass type equations of the form

$$y'(t) = -ay(t) + bf(y(t-1)) \tag{1}$$

with positive parameters a, b and a hump-shaped nonlinear $f : [0,\infty) \to [0,\infty)$; a class of nonlinear delay differential equations renowned for their rich dynamical behavior. For the nonlinearity f we assume that, in some sense, it is close to a discontinuous function $g : [0,\infty) \to [0,1]$ satisfying $g(\xi) = 0$ for $\xi > 1$. The fact that $g(\xi) = 0$ for $\xi > 1$ enables the construction of a stable periodic orbit for the equation $x'(t) = -cx(t) + dg(x(t-1))$ for some constants d and c with $d > c > 0$. This is an important step toward to the proof that Equation (1) also exhibits a stable periodic orbit, provided that the parameters a, b, and the function f are close to c, d, and g, respectively.

In this talk, we give an analytic proof for the existence of a stable periodic orbit for equation $x'(t) = -cx(t) + dg(x(t-1))$ with given $c > 0$ and a discontinuous g, provided $d > 0$ is sufficiently large.

The main example is $f(\xi) = \frac{\xi^k}{1+\xi^n}$, $k > 0$. For large n, function f is close to the function $g : [0,\infty) \to [0,1]$, where $g(\xi) = 0$ for $\xi > 1$, and $g(\xi) = \xi^k$ for $\xi \in [0,1)$.

This is a joint work with Tibor Krisztin (University of Szeged, Szeged, Hungary) and Robert Szczelina (Jagiellonian University, Kraków, Poland).

[1] G. B., T. Krisztin, R. Szczelina, Stable periodic orbits for Mackey–Glass type equations, *in preparation*