Dowker-type Theorems for Disk-Polygons in a Normed Plane

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A classical result of Dowker [1] states that for any plane convex body K in the Euclidean plane, the areas of the maximum (resp. minimum) area convex n-gons inscribed (resp. circumscribed) in K is a concave (resp. convex) sequence. It is proved that this theorem remains true if we replace area by perimeter in [4], [6] and [2], the Euclidean plane by an arbitrary normed plane [5], or convex n-gons by disk-n-gons [3], obtained as the intersection of n closed Euclidean unit disks. The aim of our paper is to investigate these problems for C-n-gons, defined as intersections of n translates of the unit disk C of a normed plane. In particular, we show that Dowker's theorem remains true for the areas and the perimeters of circumscribed C-n-gons, and the perimeters of inscribed C-n-gons. We also show that in the family of origin-symmetric plane convex bodies, for a typical element C with respect to Hausdorff distance, Dowker's theorem for the areas of inscribed C-n-gons fails.

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