Admissible Extensions of Clifford Semigroups by Inverse Semigroups

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Motivated by the desire to interpret the second partial cohomology groups in terms of normal extensions, Dokuchaev and Khrypchenko [1] introduced the notion of an admissible extension of a Clifford semigroup K by a group, and established an equivalence-preserving one-to-one correspondence between twisted partial actions of groups on K and Sieben twisted module structures on K over E-unitary inverse semigroups. We generalize the notion of admissibility to normal extensions where any inverse semigroup is allowed to appear instead of a group, and apply fundamental concepts of the structure theory of inverse semigroups (see [2]) to characterize admissible extensions of Clifford semigroups by inverse semigoups.

It is well known that each normal extension is equivalent to a normal extension associated to a pair (U, ρ) where U is an inverse semigroup and ρ is a congruence on U. We call a normal extension (U, ρ) *admissible* if $K = \operatorname{Ker} \rho$ is a Clifford semigroup, and the congruence $\rho^{\min} = \rho \cap \mu$ (where μ is the greatest idempotent-separating congruence on U) has an order preserving transversal, i.e., there exists a map $\tau \colon U/\rho^{\min} \to U$ such that $u \; \rho^{\min} \; \tau(\rho^{\min}(u))$ for every $u \in U$, and $x \leq y$ implies $\tau(x) \leq \tau(y)$ for every $x, y \in U/\rho^{\min}$.

In our characterization of admissible extensions, we use the inverse semigroup $C(U)$ of all permissible subsets of U. We associate a normal extension $(C(U, \rho), C(\rho))$ to any normal extension (U, ρ) where $C(U, \rho)$ is an inverse subsemigroup of $C(U)$ and $C(\rho)$ is a congruence on $C(U, \rho)$, and establish the following result.

Theorem 1. *A normal extension (U, ρ) is admissible if and only if the congruence $C(\rho)$ in the normal extension $(C(U, \rho), C(\rho))$ has an order preserving Billhardt transversal.*

[1] M. Dokuchaev, M. Khrypchenko, Twisted partial actions and extensions of semilattices of groups by groups, *International Journal of Algebra and Computation* **27** (2017), 887–933.

[2] M. V. Lawson, *Inverse Semigroups: The Theory of Partial Symmetries*, World Scientific, Singapore, 1998.