Ulam-Hyers Stability of Linear Differential Systems on Unbounded Intervals

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In this paper, first we give an alternate proof for the characterization of the Ulam-Hyers stability of the differential system

$$x' = A(t)x, \quad t \in \mathbb{R}, \quad x(t) \in X^n,$$

with periodic real matrix $A(t)$, where X is a Banach space. That is, it is Ulam-Hyers stable, if and only if the monodromy matrix associated to the family $\{A(t)\}_{t \geq 0}$ has no eigenvalues on the unit circle in $\mathbb{C}$. For the case when only $\mathbb{R}_+$ is considered the authors Barbu, Buşe, and Tabassum have given the same characterization in their paper [1] in 2015, where the matrix $A(t)$ is a complex matrix and $X = \mathbb{C}$.

Second we will show that the higher order differential equation

$$x^{(n)} + a_1(t)x^{(n-1)} + \ldots + a_n(t)x = 0, \quad t \in \mathbb{R}, \quad x(t) \in X^n,$$

with T-periodic real coefficients, is Ulam-Hyers stable if and only if the associated differential system

$$y' = A(t)y,$$

is Ulam-Hyers stable, where $y = (x, x', \ldots, x^{(n-1)})$. In the case of constant coefficients it was shown by the authors in 2003 [2], that the equation is Ulam-Hyers stable exactly when the characteristic polynomial has no pure imaginary roots.

These are joint results with Assoc. Prof. dr. Adriana Buică, from Babeş-Bolyai University, Faculty of Mathematics and Computer Science, Romania.

[1] Constantin Buse, Dorel Barbu, and Afshan Tabassum. Hyers-Ulam stability and exponential dichotomy of linear differential periodic systems are equivalent. *Electronic Journal of Qualitative Theory of Differential Equations*, **2015** (2015), 1–12.

[2] Takeshi Miura, Shizuro Miyajima, and Sin-Ei Takahasi. Hyers-Ulam stability of linear differential operator with constant coefficients. *Mathematische Nachrichten*, **258** (2003), 90–96.