Polymorphism-Homogeneous Groupoids on the Three-Element Set

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Previously we proved that if we assign a certain relational structure to any finite algebra in a canonical way, using solution sets of equations, then this relational structure is polymorphism-homogeneous if and only if the algebra itself is polymorphism-homogeneous [1]. Furthermore, we showed that polymorphism-homogeneity is also equivalent to the property that algebraic sets (i.e., solution sets of systems of equations) are exactly those sets of tuples that are closed under the centralizer clone of the algebra. In addition to this, we proved that the aforementioned properties hold if and only if the algebra is injective in the category of its finite subpowers. Therefore, there are several possibilities for investigating polymorphism-homogeneity. Our current goal is to determine which three-element groupoids are polymorphism-homogeneous. There are 19683 groupoids on the three-element set, but up to isomorphism their number is only 3330. In the article [2] Joel Berman and Stanley Burris investigate multiple properties of these groupoids with the help of computers. We continue this path by using a computer to help us make it clearer which groupoids are polymorphism-homogeneous. Naturally, some conjectures arise from the result given by our program(s). We focus on these conjectures, and would like to give some general condition for a groupoid to be polymorphism-homogeneous.

[1] E. Tóth and T. Waldhauser, *Polymorphism-homogeneity and universal algebraic geometry,* Discrete Mathematics & Theoretical Computer Science, special issue in honour of Maurice Pouzet **23**:(2), 2022

[2] J. Berman and S. Burris, *A computer study of 3-element groupoids,* In Logic and Algebra, (The Proceedings of the Magari Conference), 379 – 430, Marcel Dekker, Inc, 1996