Right Regular Triples of Semigroups

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In the field of abstract algebra, matrices are being researched extensively. The applications of matrices have been present in many other fields of mathematics as well, hence its a very popular topic. There are two types of matrices in the theory of semigroups which play an important role in the investigation of the structure of semigroups. They are the Rees matrices over a semigroup [3] (especially, over a group with a zero adjoined) and the right matrices of elements of a semigroup. The right matrices of elements of finite semigroups are studied in [1]. In the present paper, a connection is made between these two matrix notions.
Let $\mathcal{M}(S;\Lambda;P)$ denote a Rees $I\times\Lambda$ matrix semigroup without zero over the semigroup S, where I is a singleton, and so P is a mapping of Λ into S. Let θ_S denote the kernel of the right regular matrix representation of a semigroup S. In [2, Theorem 1], it is proved that if S is an arbitrary semigroup and P is a choice function on the collection of all θ_S-classes of S, then the factor semigroup $\mathcal{M}(S;S/\theta;P)/\theta_{\mathcal{M}(S;S/\theta;P)}$ is isomorphic to the Rees matrix semigroup $\mathcal{M}(S/\theta^*;S/\theta;P')$, where θ_S^* is the right colon congruence of θ_S, and P' is the mapping of S/θ_S onto S/θ_S^* such that $P'([s]_{\theta_S})=[s]_{\theta_S^*}$ for every $s\in S$. In other words, the semigroup of all right matrices of the Rees matrix semigroup $\mathcal{M}(S;S/\theta_S;P)$ is isomorphic to the Rees matrix semigroup $\mathcal{M}(S/\theta^*;S/\theta;P')$. On the base of this result, the following property of a sequence of semigroups is defined. A triple A,B,C of semigroups is said to be right regular, if there are mappings

$$A \xleftarrow{P} B \xrightarrow{P'} C$$

such that

$$\mathcal{M}(A;B;P)/\theta_{\mathcal{M}(A;B;P)} \cong \mathcal{M}(C;B;P').$$

In our present paper we focus on right regular triples of semigroups. By [2, Theorem 1], if A,B,C are semigroups such that $A/\theta_A\cong B$ and $B/\theta_B\cong C$, then the triple A,B,C is right regular. In [2], a right regular triple A,B,C is also given, where none of $A/\theta_A\cong B$ and $B/\theta_B\cong C$ is fulfilled. These examples motivate us to construct right regular triples of semigroups, and examine the connection between the semigroup theoretical properties of semigroups belonging to a right regular triple of semigroups.

[1] A. Nagy, Cs. Tóth, On the probability that two elements of a finite semigroup have the same right matrix, *Comment.Math.Univ.Carolin.* (to appear)

[2] A. Nagy, Cs. Tóth, On special Rees matrix semigroups over semigroups *https://doi.org/10.48550/arXiv.1609.09821 (preprint)*

[3] D. Rees, On semi-groups, *Proc. Cambridge Philosophical Society,* **36** (1940), 387-400.