Complex Bifurcation Structures in the SIRWS Model with an Asymmetric Partition of Immunity Period

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SIRS models capture transmission dynamics of infectious diseases for which immunity is not lifelong. Extending these models by a W compartment for individuals with waning immunity, the boosting of the immune system upon repeated exposure may be incorporated. This work considers the SIRWS model where a proportion of individuals experience boosting of immunity via secondary infections. Previous analyses assumed identical waning rates from R to W and from W to S. This implicitly assumes equal length for the period of full immunity and of waned immunity. We relax this restriction and allow an asymmetric partitioning of the total immune period. Stability switches of the endemic equilibrium are investigated with a combination of analytic and numerical tools. Then, continuation methods are applied to track bifurcations along the equilibrium branch. We find rich dynamics: Hopf bifurcations, endemic double bubbles, and regions of bistability. Our results highlight that the length of the period in which waning immunity can be boosted is a crucial parameter significantly influencing long-term epidemiological dynamics.

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