On Semi-Symmetric Compatible Linear Connections in Finsler Geometry

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Finsler metrics are direct generalizations of Riemannian metrics such that the quadratic Riemannian indicatrices in the tangent spaces of a manifold are replaced by more general convex bodies as unit spheres. A linear connection on the base manifold is called compatible to the Finsler metric if the induced parallel transports preserve the Finslerian length of tangent vectors. The equations describing these connections are the so-called compatibility equations. Finsler manifolds admitting compatible linear connections are called generalized Berwald manifolds. Although there are some theoretical results for the solvability of the compatibility equations, the problem is very hard to solve in general because compatible linear connections may or may not exist on a Finsler manifold and may or may not be unique. Therefore special cases are of special interest.

The case we are concerned with here is the so-called semi-symmetric compatible linear connection, whose torsion tensor can be decomposed using a 1-form on the base manifold. It was proved by Vincze ([1]) that such a connection must be unique (if it exists) and the 1-form in the decomposition of the torsion tensor can be expressed by averaging, i.e. integration of differential forms on the tangent manifold over the Finslerian indicatrices. This is an elegant theoretical result, but the integrals obtained are hard to compute in practice.

Now, we are going to present a new proof of the unicity of semi-symmetric compatible linear connections, using only basic linear algebra and some elementary properties of convex bodies. This method yields an expression of the (only possible) solution easier to compute than the original integral formulas, and we also obtain intrinsic equations (without any unknown quantities) describing necessary conditions of the solvability.

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[1] Cs. Vincze, Generalized Berwald manifolds with semi-symmetric linear connections, *Publ. Math. Debrecen* **83** (4) (2013), pp. 741–755.

[2] Cs. Vincze, M. Oláh, Finsler metrics and semi-symmetric compatible linear connections, manuscript, arXiv:2204.01034