# Flat Semigroups and Normal Surface Singularities

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The works of László and Némethi [1] provide an exact formula (up to an algorithmic term) for the Frobenius number of those numerical semigroups, which appear as semigroups associated with the local graded algebra of a weighted homogeneous complex surface singularity.

The topology of these singularities is determined by the link which is, in this case, a negative definite Seifert 3-manifold constructed by plumbing via a star-shaped dual resolution graph of the singularity. In particular, if this Seifert 3-manifold is a rational homology sphere then Pinkham[2] proves that the aforementioned semigroup is topological and can be understood with the combinatorics of the resolution graph.

The above construction is very interesting from both singularity theoretic and semigroup theoretic points of view as well, since it allows us to use the strong machinery of singularity theory in the study of numerical semigroups. Therefore it is natural to ask for the study and classification of numerical semigroups representable this way.

The main goal of this talk is to discuss some properties of the set of these representable semigroups. In particular, we consider a special family, called the flat semigroups ([3]), and we prove that they are representable by rather special star-shaped resolution graphs. Then using the properties of these graphs we get a complete description for the generators of flat semigroups and we calculate explicitly their Frobenius numbers regarding their original definition (See [2]).

This is a joint work with T. László.

[1] T. László, A. Némethi, *On the geometry of strongly flat semigroups and their generalizations*, A panorama of singularities: conference in celebration of Lê Dũng Tráng's 70th birthday, 109–136, Universidad de Sevilla, 2020.

[2] Pinkham, H., Normal surface singularities with $\mathbb{C}^*$ action, *Math. Ann.* **277** (1977), 183–193

[3] Raczunas, M. and Chrząstowski-Wachtel, P., A Diophantine problem of Frobenius in terms of the least common multiple, *Discrete Math.* **150** (1996), 347–357.