CSM – The 6th Conference of PhD Students in Mathematics

Program and Abstracts

Organized by the Bolyai Institute of the University of Szeged

June 29 – July 1, 2022
Szeged, Hungary

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Preface

Between 29th of June and 1st of July, 2022, the Bolyai Institute of the University of Szeged organizes the Conference for PhD Students in Mathematics, which is the 6th event of a conference series. The conference is held in parallel with the 13th Conference for PhD Students in Computer Science, with common invited speakers and joint social programs.

The main aim of our conference is on the one hand to provide opportunities to PhD students and to young researchers to present their field of interest and their results in 20-minute talks in English. On the other hand, students can learn about the most recent research areas from the plenary talks given by renowned invited speakers. An equally important aspect is to provide a social experience to future mathematicians.

The topics of interest of the conference are mathematics and related fields, including theory and applications, e.g. algebra, analysis, differential equations, geometry, numeric and symbolic computation, and stochastics.

The contributed talks will have a length of 20 minutes with additional 5 minutes for discussion. The participants of the conference are invited to submit new research papers for publication in the issues of *Acta Scientiarum Mathematicarum (Szeged)*.

Szeged, June 2022
Scientific and Organizing Committee

Program

Wednesday

09:00 **Registration – Bolyai Room**

09:30 **Opening – Bolyai Room**

Session: Stochastics – Bolyai Room

09:40 **Fanni K. Nedényi**
Probability Equivalent Level of Value at Risk and Higher-Order Expected Shortfalls

10:05 **Máté Szalai**
Stochastic Modeling of the Bactericidal Potency

10:30 ☕ **Coffee break**

Plenary Talk – Bolyai Room

11:00 András Benczúr
Research on Networks in the AI National Lab Hungary

Session: Algebra 1 – Bolyai Room

12:00 **Endre Tóth**
Polymorphism-Homogeneous Groupoids on the Three-Element Set

12:25 **Csaba Tóth**
Right Regular Triples of Semigroups

12:50 **Gergő Gyenizse**
Mal'tsev Conditions and Graph Powers

13:30 🍽 **Lunch - Hági Udvar (Kelemen L. street 3.)**
group photo before lunch in the building of Bolyai Institute at 13:15

Session: Algebra 2 – Bolyai Room

14:30 **Ádám Kunos**
On the Automorphism Group of the Substructure Ordering of Finite Directed Graphs

14:55 **István Mozgovoj**
On Group Algebras with Metabelian Group of Units

Session: Didactics 1 – Vályi Room (in Hungarian)

14:30 **Gábor Torma**
Teaching Mathematics During the COVID-19 Lockdown

14:55 **Judit Szilágyi**
Teaching the Basics of Artificial Intelligence at High School

15:20 **Tivadar Danka**
Modern Online Media in Mathematics Education

15:45 ☕ **Coffee break**

Session: Mathematics in Natural Science – Bolyai Room

16:05 **Andrea Nagy**
Pathway to Numerical Simulations of Stripped-Envelope Supernovae

16:30 **Péter Boldog**
Computer Efficient Stochastic Models of Cell Cultures

16:55 **Anna Barlay**
Spatio-Temporal Membrane Potential and Resistive Current Reconstruction from Parallel Multielectrode Array and Intracellular Measurements in Single Neurons

17:25 **Evans Korir**
Clustering of Countries Based on Social Contact Patterns in Epidemiological Modelling

17:50 **Marcell Dabis**
Flood Wave Detection and Representation in Time Series Data

18:15 **Bálint Batki**
Deep Learning-Based Flood Prediction of River Tisza

Session: Didactics 2 – Vályi Room (in Hungarian)

16:05 **Bence Torma**
Using Permutation Games in the Teaching of Functions

16:30 **Miklós Tekeli**
Interdisciplinary Teaching of Throws in High School

16:55 **Krisztina Horti**
On the Teaching of Exponential Functions: an Application-Based Introduction for Biology Students

19:00 🍽 **Reception – Dean's Office (Bolyai Building, ground floor)**

Thursday

Session: COVID – Bolyai Room

9:00 **Nóra Juhász**
In Silico Evaluation of Paxlovid's Pharmacometrics for SARS-CoV-2: a Multiscale Approach

9:25 **Golsa Sayyar**
Epidemic Patterns of Emerging Variants

9:50 **Saumen Barua**
A Compartmental Model for COVID-19 to Assess Effects of Non-Pharmaceutical Interventions with Emphasis on Contact-Based Quarantine

10:15 **Tamás Tekeli**
Adaptive Group Testing in a COVID-19 Model

10:40 ☕ **Coffee break**

Plenary Talk – Bolyai Room

11:00 Gergely Röst
The COVID-19 Modelling and Epidemiology Task Force in Hungary

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Session: Epidemic Models – Bolyai Room

12:00 **Bornali Das**
A Mathematical Model of Herpes and Chlamydia Co-Infection In Humans

12:25 **Ibrahim Nali**
Global Dynamics of a Within-Host Model for Usutu Virus

12:50 **Richmond Opoku-Sarkodie**
Complex Bifurcation Structures in the SIRWS Model with an Asymmetric Partition of Immunity Period

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13:30 🍽 **Lunch - Hági Udvar (Kelemen L. street 3.)**

Session: Differential Equations – Bolyai Room

14:30 **Gábor Benedek**
Periodic and Connecting Orbits for Delay Differential Equations

14:55 **Bach Ngoc Pham Le**
Nonconstant Periodic Solutions of a Nonlinear Delay Equation

15:20 **Anita Windisch**
Dynamical Behaviour of a Population-Based Neuron Model with Adaptation Current

15:45 **György Tőtős**
Ulam-Hyers Stability of Linear Differential Systems on Unbounded Intervals

16:40 **Social program**
Visiting the Votive Church (Organ presentation at 16:50, tower visiting at 17:30)

19:00 **Gala Dinner – Sótartó Halászcsárda (Roosevelt square 14.)**

Friday

Session: Geometry 1 – Bolyai Room

9:00 **Bushra Basit**
Geometric Extremum Problems in Space of Constant Curvature

9:25 **Zsolt Baja**
Flat Semigroups and Normal Surface Singularities

9:50 **Levente Szemerédi**
Higher Order Homological Percolation on the Torus

10:15 **Márk Oláh**
On Semi-Symmetric Compatible Linear Connections in Finsler Geometry

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10:40 ☕ **Coffee break**

Plenary Talk – Bolyai Room

11:00 Herbert Edelsbrunner
A Window to the Persistence of Linear Lists or Maps

Session: Graphs and Games – Bolyai Room

12:00 **Humara Khan**
Characterization of Minimally Tough Chordal Graphs with Small Toughness

12:25 **Boróka Olteán-Péter**
Interpretations of Nash Equilibrium Points

12:50 **Panna Fekete**
Sampling Lemma for Unbounded Kernels

13:30 🍽 **Lunch - Hági Udvar (Kelemen L. street 3.)**

Session: Geometry 2 – Bolyai Room

14:30 **Miklós Eper**
Elliptic Fibrations on the Rational Elliptic Surface

14:55 **Alexandra Szabó**
On the Variance of the Volume of Random Polytopes

15:20 **Kinga Nagy**
Best and Random Approximations with Generalized Disc-Polygons

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17:00 🍽 **Wine and Cheese Party – Szőkefalvi Room**

Abstracts of Plenary Talks

Research on Networks in the AI National Lab Hungary

András Benczúr
SZTAKI, Hungary

I will briefly cover the activities of the Artificial Intelligence National Laboratory Hungary, a consortium of 11 institutions in the country that cover a wide range of research in AI theory and applications.

In the main part of the presentation, I discuss recent methods to analyze and model network data accessible as a stream of edges, such as interactions in a social network service, or any other graph data with real-time updates from a stream. First I introduce the data streaming computational model and give examples of the so-called temporal networks. I describe how low-rank approximation, network embedding, link prediction, and centrality algorithms can be implemented and updated while the edge stream is processed.

I show how dynamic network analysis techniques can be used in Twitter discussions to predict centrality, identify similar users, and classify for example anti-vaxxers. As another application, I show how the network of cryptocurrency transactions leak information on the identity of the participating users.

For most part, I provide sample data and implementation as Python codes packaged in a Docker image.

A Window to the Persistence of Linear Lists or Maps

Herbert Edelsbrunner
IST, Austria

With increasing sizes of data, there is also an increase in the number and size of maps that describe aspects of data. This talk will give a systematic answer to the question which features of a continuous maps are how important. For real-valued maps on an interval or a circle, we get a hierarchical organization of the features with elementary means and efficient algorithm that dynamically maintain this hierarchy with path-decomposed ordered binary trees. We mention here that this approach generalizes to higherdimensional maps and is used under the name of persistent homology in topological data analysis. This more general view of the subject will not appear during the main part of the talk but can be discussed afterwards if there is interest.

The mathematical work is based on joint research with Ranita Biswas, Sebastiano Cultrera, Morteza Saghafian, and the algorithmic work on joint research with Sebastiano Cultrera, Monika Henzinger, and Wolfgang Ost.

The COVID-19 Modelling and Epidemiology Task Force in Hungary

Gergely Röst
University of Szeged, Szeged, Hungary

The Hungarian COVID-19 Mathematical Modelling and Epidemiological Analysis Task Force (also known as the "epimath team") was assembled in the early phase of the pandemic in March 2020, to provide in-depth epidemiological situation reports, forecasting, and scenario analysis to support evidence informed decision making during the COVID-19 pandemic. It is a multidisciplinary team of specialists from many institutions across the country, including mathematicians, medical doctors, epidemiologists, statisticians, network scientists, system biologists, public health experts, computer scientists and mathematical social scientists. Our aim was to integrate a wide range of competencies to tackle the complex public health, economical and societal challenges posed by the pandemic. This was an innovative initiative in Hungary, and in this talk we summarize how this team has worked and what has been achieved in the past two years, regarding policy advisory and scientific research. Several specific examples will be highlighted from the past pandemic waves to illustrate how the task force contributed to the fight against SARS-COV-2.

Abstracts of Contributed Talks

Flat Semigroups and Normal Surface Singularities

Zsolt Baja
Babeş–Bolyai University, Cluj-Napoca

The works of László and Némethi [1] provide an exact formula (up to an algorithmic term) for the Frobenius number of those numerical semigroups, which appear as semigroups associated with the local graded algebra of a weighted homogeneous complex surface singularity.

The topology of these singularities is determined by the link which is, in this case, a negative definite Seifert 3-manifold constructed by plumbing via a star-shaped dual resolution graph of the singularity. In particular, if this Seifert 3-manifold is a rational homology sphere then Pinkham[2] proves that the aforementioned semigroup is topological and can be understood with the combinatorics of the resolution graph.

The above construction is very interesting from both singularity theoretic and semigroup theoretic points of view as well, since it allows us to use the strong machinery of singularity theory in the study of numerical semigroups. Therefore it is natural to ask for the study and classification of numerical semigroups representable this way.

The main goal of this talk is to discuss some properties of the set of these representable semigroups. In particular, we consider a special family, called the flat semigroups ([3]), and we prove that they are representable by rather special star-shaped resolution graphs. Then using the properties of these graphs we get a complete description for the generators of flat semigroups and we calculate explicitly their Frobenius numbers regarding their original definition (See [2]).

This is a joint work with T. László.

[1] T. László, A. Némethi, *On the geometry of strongly flat semigroups and their generalizations*, A panorama of singularities: conference in celebration of Lê Dũng Tráng's 70th birthday, 109–136, Universidad de Sevilla, 2020.

[2] Pinkham, H., Normal surface singularities with $\mathbb{C}^*$ action, *Math. Ann.* **277** (1977), 183–193

[3] Raczunas, M. and Chrząstowski-Wachtel, P., A Diophantine problem of Frobenius in terms of the least common multiple, *Discrete Math.* **150** (1996), 347–357.

Spatio-Temporal Membrane Potential and Resistive Current Reconstruction from Parallel Multielectrode Array and Intracellular Measurements in Single Neurons

Zoltán Somogyvári, **Anna Barlay**, Domokos Meszéna, Péter Boldog, Dorottya Cserpán, Kinga Tóth, Lucia Wittner, István Ulbert
Wigner Research Centre of Physics, Budapest, Hungary

Here we show that we have developed a novel mathematical data analysis method to determine spatio-temporal distribution of neuronal membrane potentials and the distribution of membrane current components during brain activity. My interdisciplinary presentation will incorporate a unique mathematical model, a software programming solution, and an innovative measurement procedure as well. Our model is based on the inverse solution of Poisson equation, via calculating the spatio-temporal solution of partial differential equations from one-dimensional voltage dataset. Based on parallel multichannel extracellular and single-channel intracellular potential recordings, it is possible to reconstruct the spatio-temporal distribution of membrane potential with the spatial resolution of the extracellular recordings in a single neuron. Moreover, we show, that reconstruction of intracellular membrane potential made possible the distinction between two components of the current source density (CSD): the resistive and the capacitive currents. This distinction would provide a clue to the proper interpretation of the CSD distribution, as the resistive component corresponds to the active channel currents, both synaptic and voltage-sensitive channel membrane currents, while capacitive current corresponds to the passive counter currents. The importance of this distinction is further emphasized by different features of the resistive membrane current distribution compared to the CSD. As the CSD is a net membrane current, the sum of the CSD along a whole intact cell should be zero at each time moment, according to the charge conservation law. In contrast to this, the sum of the resistive current should not be necessarily zero since it governs the membrane potential dynamics. Thus, estimation of the spatial distribution of the resistive membrane current makes possible the distinction between active and passive sinks and sources of the CSD map and localization of the synaptic input currents, which makes the neuron fire. I am going to validate our reconstruction approach on simulations and demonstrate its application on simultaneous and co-localized rat brain slice recordings. I am going to show our results of the digital spatio-temporal reconstruction. The algorithm has more dynamic purposes as well.

[1] A. L. Hodgkin, A. F. Huxley, A quantitative description of membrane current and its application to conduction and excitation in nerve. *The journal physiology* **117** (1952), 500–544.

[2] G. Buzsáki, Large-scale recording of neuronal ensembles. *Nature neuroscience 7,* **no. 5** (2004): 446-451.

[3] K. H. Pettersen, H. Lindén, A. M. Dale, G. T. Einevoll, Extracellular spikes and CSD. *Handbook of neural activity measurement* **1** (2012): 92-135.

[4] G. T. Einevoll, C. Kayser, N. K. Logothetis, S. Panzeri , Modelling and analysis of local field potentials for studying the function of cortical circuits. *Nat. reviews neuroscience 14,* **770** (2013).

[5] Z. Somogyvári, D. CSerpán, I. Ulbert, P. Erdi Localization of single-cell current sources based on extracellular potential patterns: the spike csd method. *Eur. J. neuroscience* **36** (2012): 3299–3313.

[6] K. H. Pettersen, A. Devor, I. Ulbert, A. M. Dale, G. T. Einevoll, Current-source density estimation based on inversion of electrostatic forward solution: effects of finite extent of neuronal activity and conductivity discontinuities. *Journal of neuroscience methods* **154** no. 1-2 (2006): 116-133.

[7] T. V. Ness, C. Chintaluri, J. Potworowski, S. Łęski, H. Głąbska, D.K. Wójcik, G. T. Einevoll, Modelling and analysis of electrical potentials recorded in microelectrode arrays (MEAs). *Neuroinformatics* **13** (2015): 403–426.

[8] F. Hamilton, T. Berry, T. Sauer, Tracking intracellular dynamics through extracellular measurements. *PloS one* **13** no. 10 (2018): e0205031.

[9] D. Cserpán, D. Meszéna, L. Wittner, K. Tóth, I. Ulbert, Z. Somogyvári, D. K. Wójcik, Revealing the distribution of transmembrane currents along the dendritic tree of a neuron from extracellular recordings. *Elife* **6** (2017): e29384.

[10] Z. Somogyvári, L. Zalányi, I. Ulbert, P. Erdi, Model-based source localization of extracellular action potentials. *J. neuroscience methods* **147** (2005): 126–137.

A Compartmental Model for COVID-19 to Assess Effects of Non-Pharmaceutical Interventions with Emphasis on Contact-Based Quarantine

Saumen Barua, Bornali Das, Attila Dénes
Bolyai Institute,University of Szeged, Szeged, Hungary

Relative to the number of casualties, COVID-19 ranks among the ten most devastating plagues in history. The pandemic hit the South Asian nation of Bangladesh during the early March, 2020, and has greatly impacted the socio-economic status of the country [1]. We propose a compartmental model for COVID-19 dynamics, introducing a separate class for quarantined susceptibles, synonymous to isolation of individuals who have been exposed and are suspected of being infected. Following [2] the current model assumes a perfect quarantine based on contacts with infectious individuals and concentrating on the non-pharmaceutic interventions applicable at the beginning of a new pandemic, includes some features which make it different from the great majority of COVID models. Numerical simulation is conducted to investigate the efficiency of disease control by segregating suspected individuals and other non-pharmaceutical interventions. In addition, we assort quantitatively the importance of parameters that influence the dynamics of the system. Fitting the system to the early phase of COVID-19 outbreaks in Bangladesh, by taking into account the cumulative number of cases with the data of the first 17-week period, the basic reproduction number is estimated as 1.69.

[1] https://covid19.who.int/region/searo/country/bd

[2] M. Lipsitch, T. Cohen, B. Cooper, J. M. Robins, S. Ma, L. James, G. Gopalakrishna, S. K. Chew, C. C. Tan, M. H. Samore, et al., Transmission dynamics and control of severe acute respiratory syndrome, *Science* **300** (5627) (2003) 1966–1970.

Deep Learning-Based Flood Prediction of River Tisza

Bálint Batki, Luca Rátki, Zsolt Vizi
Centre for Energy Research, Budapest, Hungary

Precise prediction of flood level of river Tisza can save lives and valuable resources. The current forecasting methodology, implemented at the water management directorates, is based on physical models and expert judgements. However, historical data on daily water levels along the river are available and provide an opportunity for a data-driven solution.

Our new data-driven solution is based on deep learning models aiming to recognize complex patterns in past observations. We focus on the water level prediction at Szeged and present our results using two state-of-the-art model architectures: encoder-decoder type recurrent neural network (LSTM-LSTM) and Temporal Fusion Transformer (TFT). Since temporal component of the data requires advanced concepts in the architectures, we elaborate and develop the models from aspect of interpretability, which relies on mathematical foundations of time series.

The presented results are from an ongoing collaboration between the Bolyai Institute (University of Szeged) and the regional water management directorate (ATIVIZIG) and a joint work with Luca Rátki and Zsolt Vizi from the Bolyai Institute.

Periodic and Connecting Orbits for Delay Differential Equations

Gábor Benedek
University of Szeged, Bolyai Institute, Szeged, Hungary

We consider a class of nonlinear differential equations with delayed feedback. It is well known that these can generate complex or even chaotic dynamics. One such equation is the *Mackey-Glass equation*. An article by Mackey and Glass in 1977 ([3]) studies physiological processes in which time lag plays a significant role. The model they prescribe is based on the fact that it takes a significant amount of time from the formation of blood cells to their entry into the bloodstream. The parameters occurring in the model result in varied, often chaotic behaviour of the system, so understanding dynamics is an area still researched today.

In [1] we considered a more general form of the Mackey-Glass equation, which is suitable for modeling a significant Allee effect in population dynamics. Our results relate to the existence of orbitally asymptotically stable periodic and certain heteroclinic orbits to which we apply the general theory of infinite dimensional dynamics systems developed in [2]. The method of the [1] article can be extended to more general nonlinearities that also allows the Allee effect. In the case of the Allee effect, a new unstable equilibrium occurs that is not present in the Mackey-Glass equation.

In the talk we summarize the previous results and describe those conditions that define such a nonlinear feedback function class for which our results still hold. Additionally we delineate our present research goals about understanding the dynamics of the general case.

This is a joint work with **Tibor Krisztin**, University of Szeged, Bolyai Institute.

[1] Gábor Benedek, Periodic and connecting orbits for Mackey-Glass type differential-delay equations, *in preparation.*

[2] Tibor Krisztin, Hans-Otto Walther, Jianhong Wu, Shape, Smoothness and Invariant Stratification of an Attracting Set for Delayed Monotone Positive Feedback, *Fields Institute Monographs, Vol. 11, Amer. Math. Soc., Providence, RI,* (1999).

[3] M. Mackey and L. Glass, Oscillation and chaos in physiological control systems, *Science, New Series*, **197** (1977), 286-289.

Computer Efficient Stochastic Models of Cell Cultures

Péter Boldog
University of Szeged, Szeged, Hungary

With the emergence of personalised medical therapies, digital pharmacokinetics and pharmacodynamics, mathematical modelling of cell cultures has become a prominent area of *in silico* biology. In this talk, we will address the modelling issues of cell movement and division, with a special focus on the phenomenon of volume exclusion.

Consider circular cells with uniform size of diameter d and place the cells on a 2 dimensional square lattice with grid constant d. Under these conditions, the principle of volume exclusion states that there can be at most one cell per lattice site.

Our goal is to provide a sound interpretation of volume exclusion and exact stochastic algorithms for the numerical simulation of cell cultures in continuous time and discrete space. We present the *prompt decision method*, which applies the exclusion principle directly, and the *reduced rate method*, which applies it indirectly. Then we prove that the two methods are statistically equivalent in the sense that in a given state of the system the time until the next event and also the next event to be realized are drawn from the same time and event distribution in case of both methods.

Finally, we give a condition that we can use to decide which one of the above algorithms is the fastest in a given state of the simulation, thus we can minimize the simulation runtime.

Geometric Extremum Problems in Space of Constant Curvature

Bushra Basit, Zsolt Lángi
Budapest University of Technology and Economics, Budapest, Hungary

Abstract: It was proved by Boroczky [1] and Peyerimhoff [2] that among simplices inscribed in a ball in spherical and hyperbolic space, respectively, the regular simplices have maximal volume. In this lecture we show that among simplices circumscribed about a ball in hyperbolic space, the regular simplices have minimal volume. We also investigate analogous questions for d-dimensional spherical and hyperbolic polytopes with $d+2$ vertices.

[1] K. Böröczky, *On an extremum property of the regular simplex in S^d*, Colloq. Math. Soc. János Bolyai **48**, (1987), 117-121.

[2] N. Peyerimhoff, *Simplices of maximal volume or minimal total edge length in hyperbolic space*, J. Lond. Math. Soc. **66** (2002), 753-768.

Flood Wave Detection and Representation in Time Series Data

Marcell Dabis, Zsolt Vizi
University of Szeged, Szeged, Hungary

In this talk we will consider a flood wave as consecutive peaking in water levels alongside the measuring points of the river. This implies that we need to check the data of the measuring points sorted by river kilometer, and look for peaks that are close to each other geologically and in time as well. From the flood wave's propagation time, and overall length, we can derive useful statistics for river control.
In this work a graph representation of the flood wave's propagation will be presented. We will cover the problems/topics of the pattern of a flood wave and also the merging of flood waves. This research is the part of the currently ongoing cooperation between the Bolyai Institute and the water directorate of Szeged (ATIVIZIG).
Keywords: pattern recognition, flood wave, depth-first search, graph representation

Modern Online Media in Mathematics Education

Tivadar Danka

Twitter, Facebook, LinkedIn, and YouTube – these online social media platforms are extremely underrated and underused in education. However, they have millions of users logging on daily to consume technical and educational content, and their demand is significantly higher than the supply.

Social media as an educational content platform first became prevalent in the computer science community, and this type of application slowly trickled down from the academic community to the industry. Nowadays, the impact of academic papers is often measured by the buzz it generates on social media (an idea called "altmetrics"), and companies invest millions of dollars to build profiles and attract attention. During this process, e-learning has skyrocketed, and in some industries, private online education has become a serious competitor for the traditional academic system.

In the last two years, I built a successful online brand with high-quality educational mathematics content that reaches millions of students every month, one that turned into a full-time job for me. This talk is going to be about the lessons I learned along the way, and the massive potential of online media for us mathematicians, teachers, and educators.

A Mathematical Model of Herpes and Chlamydia Co-infection In Humans

Bornali Das, Gergely Röst
University of Szeged, Szeged, Hungary

A mathematical model was formulated to investigate the co-interaction of *Chlamydia trachomatis* (*C. trachomatis*) and herpes simplex virus (HSV) in human beings; taking into account the HSV-induced persistence in *C. trachomatis*. The *C. trachomatis* only and the HSV only sub-systems are first explored, whereby analytic expressions for the threshold parameters are obtained in each sub-model and the global dynamics are investigated. Global dynamics of the co-infection shows that the infections die out whenever the threshold parameters for the respective sub-systems and co-infection model is less than unity. On the contrary, *C. trachomatis* cannot invade the population provided that the herpes reproduction number is larger than unity. We further investigate the impact of HSV in the *C. trachomatis* stable system, and vice versa. The analysis shows that herpes invades uniformly in an entirely susceptible population as well as in a *C. trachomatis* stable population. However, the analysis of the reverse case demonstrates the two infection co-exists only when the reproduction numbers of *C. trachomatis* computed at the HSV equilibrium exceeds unity. The theory of asymptotically autonomous systems with limiting systems is used to show the existence of an endemic equilibrium when all the reproduction numbers are larger than unity, where all the state variables are positive. Numerical simulations are carried out to graphically illustrate the asymptotic stability of the different equilibria of the system and the existence of a positive endemic equilibrium.

Elliptic Fibrations on the Rational Elliptic Surface

Miklós Eper, Szilárd Szabó
Budapest University of Technology and Economics, Budapest, Hungary

In 1963 K. Kodaira [2] gave the complete list of singular fibers, which can occur in elliptic fibrations. The question, which combination of these singular fibers are possible on the rational elliptic surface, have been answered by U. Persson and R. Miranda [3] in 1990. An interesting feature of this topic is, that elliptic fibrations on the rational elliptic surface can be considered from algebraic geomertic point of view as well. Elliptic fibrations with certain types of singular fibers can be constructed via blowing up pencils on the rational elliptic surface.

All fibrations in our studies arise from blow up of pencils, and this is the process we will use to construct such fibrations. These constructions of elliptic fibrations with certain types of singular fibers have been described in the papers, provided by A. Stipsicz, Sz. Szabó and P. Ivanics, except the case of fibration with singular fiber I_1^*. The main result of our investigation is the explicit, algebraic geometric construction of the 13 possible configurations of elliptic fibrations with I_1^* fiber. These constructions on the Hirzebruch surface, and the pencils from which they arise, have not been described yet.

We introduce the most important definitions and theorems related to elliptic fibrations on the rational surface, the main literature used here is the book *Principles of algebraic geometry* by P. Griffiths and J. Harris [1]. We also describe, which combinations can occure with an I_1^* type fiber, based on the paper of A. Stipsicz, Z. Szabó, and Á. Szilárd [4]. Finally, we present our results, i.e. the constructions of rational elliptic surfaces, containing the 13 possible singular fiber configurations, by choosing the right pencils, and supporting our calculations with figures, illustrating the process.

[1] Griffiths, P., Harris J. Principles of Algebraic Geomet- ry, Wiley., (1994)

[2] Kodaira, K. On compact analytic surfaces: II, Ann. Math., 77, (1963), 563-626.

[3] Miranda, R. Persson's list of singular fibers for a rational elliptic surface, Math. Z., 205, (1990), 191-211.

[4] Stipsicz, A.I., Szabó , Z., Szilárd A. Singular fibers in elliptic fibrations on the rational elliptic surface, *Periodica Mathematica Hungarica*, (2007), 54:137–162

Sampling Lemma for Unbounded Kernels

Panna Tímea Fekete, Dávid Kunszenti-Kovács
Eötvös Loránd University, Budapest, Hungary
and Alfréd Rényi Institute of Mathematics, Budapest, Hungary

Graph sequences with limits that can be represented by an unbounded, L^p kernel for some $p < \infty$ arise naturally from various random graph models, among others the dense Preferential Attachment Graph random multigraph model. Whilst it was shown that even for unbounded kernels W one has probability 1 convergence in density of the W-random graph sequence $\mathbb{G}(W, n)$ to W itself, this does not imply the same convergence in cut distance. Here we first present high probability bounds on the closeness of the cut norms of the kernel and its samples, i.e., a First Sampling lemma for unbounded kernels.

For some fixed $k \in \mathbb{Z}_+$ let X be a random ordered k-subset of $[0,1]$. Let $U[X]$ denote the step-function on $[0,1]^2$ with uniform steps of length $1/k$ in each variable, and values given by $(U(X_i, X_j))_{i \neq j \in [k]}$, and 0 on the main diagonal.

Given an $n \times n$ matrix A and a function $U \in L^1([0,1]^2)$, their cut norm is:

$$\|A\|_\square := \frac{1}{n^2} \max_{S,T \subseteq [n]} \left| \sum_{i \in S, j \in T} A_{ij} \right| \quad \text{and} \quad \|U\|_\square := \sup_{S,T \subseteq [0,1]} \left| \int_{S \times T} U(x,y)dxdy \right|.$$

Our goal is to provide a high probability bound on the difference of the norm $\left\|U[X]\right\|_\square$ of the random sample and the original norm $\left\|U\right\|_\square$. The typical application would be to check with very high probability that $\|U\|_\square$ is small, via looking at the samples.

Our main result is a generalization of the graphon sampling lemma ([1], [2]):

Theorem 1. *(First Sampling Lemma for Unbounded Kernels). Let $\gamma > 0$ and $m > 4$ be constants such that $m\gamma > 2$. Then for any function $U \in L^m_{sym}([0,1]^2)$, there exist constants $C, C_0, C_1 > 0$ only depending on γ, m and $\|U\|_m$ such that with probability at least $1 - C_1 k^{2-\gamma m}$,*

$$-C_0 k^{-1/2+\gamma} (\ln k)^{1/2} \leq \left\|U[X]\right\|_\square - \left\|U\right\|_\square \leq C \cdot k^{-1/4+\gamma/2} (\ln k)^{1/4}.$$

Thus we got that under mild conditions the bound is arbitrarily close in order to the bound for L^∞ graphons, but at the cost of a much larger, only polynomially small (as opposed to exponentially small) exceptional event set. We note, however, that such a polynomial sized exceptional set is still adequate for most applications.

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[1] N. Alon, W. Fernandez de la Vega, R. Kannan and M. Karpinski, Random sampling and approximation of MAX-CSPs, *J. Comput. System Sci.* **67** (2003), 212–243.

[2] C. Borgs, J.T. Chayes, L. Lovász, V.T. Sós and K. Vesztergombi, Convergent Graph Sequences I: Subgraph frequencies, metric properties, and testing, *Advances in Math.* **219** (2008), 1801–1851.

Mal'tsev Conditions and Graph Powers

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Let $\mathcal{I}$ be a finite set of identities over any signature. Then $\mathcal{I}$ induces a *strong Mal'tsev class* denoted by $\mathfrak{M}(\mathcal{I})$: an algebra is in this class precisely if it has terms that satisfy $\mathcal{I}$. For example, if

$$\mathcal{I} = \{a+b \approx b+a, (a+b)+c \approx a+(b+c), a-a \approx 0, a+0 \approx a, a-b \approx a+(0-b),\\ a\star(b+c) \approx a\star b + a\star c, (b+c)\star a \approx b\star a + c\star a, (a\star b)\star c \approx a\star(b\star c)\},$$

and

$$\mathcal{J} = \{a+b \approx b+a, (a+b)+c \approx a+(b+c), a-a \approx 0, a+0 \approx a, a-b \approx a+(0-b),\\ a\star(b+c) \approx a\star b+a\star c, (b+c)\star a \approx b\star a+c\star a, a\star a \approx 0, (a\star b)\star c+b\star(c\star a)+c\star(a\star b)\},$$

then an algebra is in $\mathfrak{M}(\mathcal{I})$ and $\mathfrak{M}(\mathcal{J})$ if it has ring terms and Lie-ring terms, respectively. Somewhat surprisingly, these classes are the same: a ring becomes a Lie-ring if we change the multiplication to the commutator operation defined by $[a,b] := ab - ba$, also, a Lie-ring becomes a ring if we change the Lie-bracket into the the operation defined by $ab := 0$.

It can be very hard to decide whether two sets of identities generate the same strong Mal'tsev class, or whether they contain each other. A related question is whether such a class $\mathfrak{K}$ is *prime*: is it true that if $\mathfrak{K}$ contains $\mathfrak{L}_1 \cap \mathfrak{L}_2$ then it contains either $\mathfrak{L}_1$ or $\mathfrak{L}_2$?

We have answered this question affirmatively in the case when $\mathfrak{K}$ is the class of algebras that generate a congruence permutable variety (induced by the identities $m(x,x,y) \approx m(y,x,x) \approx y$). While this is a purely algebraic result, the proof uses exponentiation of graphs. For (directed or undirected) graphs $\mathbb{G}$ and $\mathbb{H}$ the vertices of the graph $\mathbb{G}^{\mathbb{H}}$ are the mappings $H \to G$, and the edge relation is defined by

$$f_1 \to f_2 \;\Leftrightarrow\; \forall\, h_1 \to h_2 :\; f_1(h_1) \to f_2(h_2).$$

The main combinatoric ingredient to our algebraic result is the following simple statement:

Theorem 1. *If $\mathbb{G}$ and $\mathbb{H}$ are graphs such that neither if complete, but each has a vertex that is a neighbor of any vertex (including itself), then there is a nonempty graph $\mathbb{K}$ so that $\mathbb{G}^{\mathbb{K}}$ and $\mathbb{H}^{\mathbb{K}}$ are isomorphic.*

On the Teaching of Exponential Functions: an Application-Based Introduction for Biology Students

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Powers, exponential, and logarithmic functions are essential in the natural sciences. They serve as models of many phenomena, and therefore play a key role in the teaching of subjects and mathematics. However, after analyzing five mathematics textbooks written for 11th graders, we concluded that biology as a motivation hardly appears in the teaching of exponential functions. So, don't be surprised that students interested in biology don't like math. Many people ask why they need to learn math if they won't use it later anyway. As our teaching experience in both high school and university (biology, pharmacist) shows, and a survey we did also demonstrates that a lot of people are afraid to learn math, not only in college, but also during their high school years. It is interesting to note that the little ones are not yet afraid... Learning the exponential and logarithmic function is especially problematic and difficult, although they are naturally connected with several biological processes.

In our courses for biologists and pharmacy students, we show a number of practical examples, illustrate the theoretical concepts with applications immediately. Instead of abstract discussion, we often focus on interactive graphical illustration. Based on our experience, we have developed a visual introduction to exponential and logarithmic functions for students with a biological interest, including some deeper topics for advanced students. Introductory examples help the development of concepts, definitions, and applications as well, such as the tricky behavior of exponential changes (slow at first, very fast later). Although the time of logarithmic rulers is over, the importance of logarithmic plots is unquestionable; the study of exponential functions is much easier with them. Exponential equations mean particularly difficult problem the student. The mathematics curriculum is limited to their technical handling (e.g., transforming to quadratic equations), but biological processes are not aware of this. Indeed, students need to know that most of the equations cannot be solved symbolically... and the curriculum should suggest what to do in these cases (e.g. approximation methods), even if full-depth discussion is not possible.

[1] K. Horti, Teaching exponential and logarithmic functions in high schools with biology motivation, master's thesis (in Hungarian), *Bolyai Institute, University of Szeged* (2020)

In Silico Evaluation of Paxlovid's Pharmacometrics for SARS-CoV-2: a Multiscale Approach

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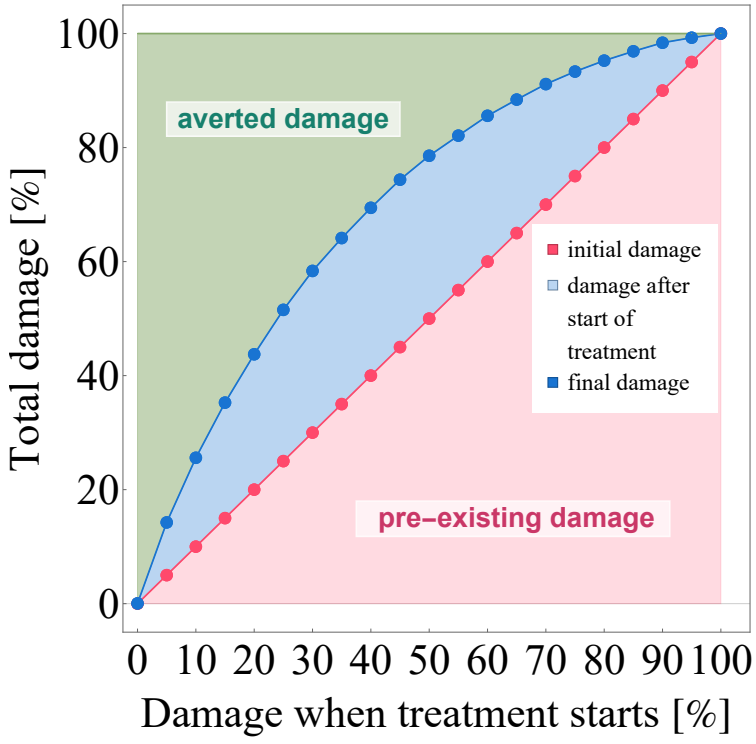
Paxlovid is a promising, orally bioavailable novel drug for SARS–CoV–2 with excellent safety profiles. Our main goal here is to explore the pharmacometric features of this new antiviral. To provide a detailed assessment of Paxlovid, we propose a hybrid multiscale mathematical approach. We demonstrate that the results of the present *in silico* evaluation match the clinical expectations remarkably well: on the one hand, our computations successfully replicate the outcome of an actual *in vitro* experiment; on the other hand we verify both the sufficiency and the necessity of Paxlovid's two main components (nirmatrelvir and ritonavir) for a simplified *in vivo* case. Moreover, in the simulated context of our computational framework we visualize the importance of early interventions, and identify the time window where a unit–length delay causes the highest level of tissue damage. Finally, the results' sensitivity to the diffusion coefficient of the virus is explored in details. *Joint work with Ferenc Bartha, Sadegh Marzban, Renji Han and Gergely Röst.*



[1] F. Bartha et al., In silico evaluation of Paxlovid's pharmacometrics for SARS-CoV-2: a multiscale approach, *Viruses* **14(5), 1103** (2022); https://doi.org/10.3390/v14051103

Characterization of Minimally Tough Chordal Graphs with Small Toughness

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Let t be a real number. A graph is called *t-tough* if the removal of any vertex set S that disconnects the graph leaves at most $|S|/t$ components. The toughness of a graph is the largest t for which the graph is t-tough. A graph is minimally t-tough if the toughness of the graph is t and the deletion of any edge from the graph decreases the toughness. A graph is *chordal* if it does not contain an induced cycle of length at least 4. We will call a graph a *TT-graph* if it can be obtained from a tree of maximum degree $\Delta \geq 3$ by removing some (or all) of its vertices with degree 3 whose neighbors have degree Δ, and joining these neighbors with triangle.

Kriesell's conjectured [1] that every minimally 1-tough graph has a vertex of degree 2. This conjecture can be naturally generalized: every minimally t-tough graph has a vertex of degree $\lceil 2t \rceil$. Gyula Y. Katona and Kitti Varga [3], showed that the conjecture is true for chordal graphs when $1/2 < t \leq 1$.

In this paper we show that the Generalized Kriesell's Conjecture for chordal graphs with toughness $\leq 1/2$ by giving a characterization of such graphs. We show that for $t \leq 1/2$ a chordal graph is minimally t-tough if and only if it is a TT-graph. As a corollary, a characterization of minimally t-tough interval graphs is obtained for $t \leq 1/2$, as well.

One of the main tools in the proof is the well known representation of chordal graphs using clique trees [2]. The other important tool is general necessary and sufficient condition for minimally tough graphs given in [4].

[1] Mathias Kriesell, *In: Ed. T. Kaiser, Problems from the Workshop on dominating cycles*, Hájek, Czech Republic, 2003.
`http://iti.zcu.cz/history/2003/Hajek/problems/hajek-problems.ps`

[2] C. G. Lekkerkerker and J. Ch. Boland, *Representation of a finite graph by a set of intervals on the real line*, Fund. Math., 51 (1962), pp. 45–64.

[3] Gyula Y. Katona, Kitti Varga, *Minimally toughness in special graph classes,* arXiv:1802.00055 [math.CO] 2018.

[4] Clément Dallard, Blas Fernández, Gyula Y. Katona, Martin Milanič, *Conditions for minimally tough graphs*, manuscript 2021.

Clustering of Countries Based on Social Contact Patterns in Epidemiological Modelling

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Mathematical models have been used to understand the spread patterns of infectious diseases such as Coronavirus Disease 2019 (COVID-19). The transmission component of the models can be modelled in an age-dependent manner via introducing contact matrix for the population, which describes the contact rates between age groups. Since social contact patterns vary from country to country, we can compare and group the countries using the corresponding contact matrices.

In this talk, we will introduce a methodology, which applies clustering on the European countries represented by the *standardized* contact matrix estimated in [1] where standardization is based on epidemiological modelling. In the proposed framework we can easily change e.g. the target metric for the standardization and the distance measure to be more customized for the application area. We will demonstrate the algorithm using COVID-19 model from [2].

Keywords: age-dependent epidemic model, social contact pattern, clustering, reproduction number

[1] Prem K, Zandvoort KV, Klepac P, Eggo RM, Davies NG, Centre for the Mathematical Modelling of Infectious Diseases COVID-19 Working Group, Cook AR, Jit M. Projecting contact matrices in 177 geographical regions: an update and comparison with empirical data for the COVID-19 era. PLoS computational biology. 2021 Jul 26;17(7):e1009098.

[2] Röst G, Bartha FA, Bogya N, Boldog P, Dénes A, Ferenci T, Horváth KJ, Juhász A, Nagy C, Tekeli T, Vizi Z. Early phase of the COVID-19 outbreak in Hungary and post-lockdown scenarios. Viruses. 2020 Jul;12(7):708.

On the Automorphism Group of the Substructure Ordering of Finite Directed Graphs

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We investigate the automorphisms, i. e. symmetries, of a concrete partially ordered set, shortly poset. The elements of the poset in question are finite directed graphs and what gives the order relation between them is some sort of containment. Let $\mathcal{D}$ denote the set of (isomorphism types of) finite directed graphs, and, for $G, G' \in \mathcal{D}$, let $G \sqsubseteq G'$ denote the fact that G is (isomorphic to) a spanned subgraph of G'. The partial order we are investigating is $(\mathcal{D}; \sqsubseteq)$. In [1], the second author formulated a conjecture for the automorphism group of $(\mathcal{D}; \sqsubseteq)$. This conjecture was not at all placed on a firm footing. In this work, though still unable to prove it, we give it a firm footing with the use of some computer calculations.

It is easy to see that $(\mathcal{D}; \sqsubseteq)$ is a graded poset, that is it splits into levels the natural way. On the n-th level, we find the digraphs having n vertices. In [1], it is shown that the action of an automorphism on the first twelve levels determines it wholly. At first sight, this seems to reduce our infinite problem (of finding all the automorphisms) to a finite one. This is where computer calculations come into play as finite calculations can be done with a fast enough computer. The problem is that we can't even get close to the computational capacity that would be needed here. Nevertheless, we show that on the first few levels the automorphism group behaves according to the conjecture.

Let $\operatorname{Aut} P$ denote the automorphism group of the poset P. For a graded poset P, let P_n denote the subposet of the first n levels. It is clear that P_n is invariant under $\operatorname{Aut} P$. It turns out that we cannot really expect $\operatorname{Aut} \mathcal{D} \cong \operatorname{Aut} \mathcal{D}_n$ for any concrete n. For example we show that

$$|\operatorname{Aut} \mathcal{D}_3| \approx 1.67 \cdot 10^{13}, \tag{1}$$

which is way too big as the group of the conjecture has 768 elements. This forces us to do the following. Let $\varphi|_n$ denote the restriction of $\varphi \in \operatorname{Aut} P$ to P_n. Furthermore, let $\operatorname{Aut}_n P$ denote $\{\varphi|_n \in \operatorname{Aut} P_n : \varphi \in \operatorname{Aut} P\}$, which is clearly a subgroup of $\operatorname{Aut} P_n$. With these notations, what we show is that, in sharp contrast to (1), $\operatorname{Aut}_3 \mathcal{D}_4$ is isomorphic to the 768-element group of our conjecture, meaning that the automorphism group works on the first three levels as expected by the conjecture.

[1] Á. Kunos, Definability in the substructure ordering of finite directed graphs, *Order* **38** (2021), 401–420.

Probability Equivalent Level of Value at Risk and Higher-Order Expected Shortfalls

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The Fundamental Review of the Trading Book (FRTB) was introduced by the Basel Committee on Banking Supervision in the years following the Global Financial Crisis of 2007-2009. FRTB is expected to make a complete revision of the approach to calculating risk-based capital requirements for investments. Currently, banks are required to report their risk calculated as the 99% Value at Risk. Consider a random variable X which is the loss on a certain portfolio. Then, the Value at Risk ($\mathtt{VaR}$) of X at a level $p \in [0,1]$ is defined by

$$\mathtt{VaR}_X(p) := \inf\{x \in \mathbb{R} : F_X(x) \geqslant p\},$$

with the convention $\inf \emptyset := \infty$. Then, roughly speaking, $\mathtt{VaR}_X(0.99)$ is the loss that is likely to be exceeded only 1% of the time. While $\mathtt{VaR}$ is widely used and easy to compute, it has no information on the magnitude of the biggest 1% of losses. Moreover, it is not a coherent risk measure. Indeed, it is not subadditive, which means that $\mathtt{VaR}$ of a portfolio can be higher than the sum of $\mathtt{VaR}$s of the individual assets in the portfolio.

By FRTB, banks will soon have to switch to Expected Shortfall ($\mathtt{ES}$) at the level 0.975 instead of $\mathtt{VaR}$ at the level 0.99 for the bank-wide internal models to determine market risk capital requirements. We are interested in studying the possibility to switch to higher-order Expected Shortfalls instead. Let X be a random variable such that $X \in L^1$, and let $n \in \mathbb{N}$. The n^{th}-order $\mathtt{ES}$ of X at a level $p \in [0,1)$ is defined by

$$\mathtt{ES}_{X,n}(p) := \frac{n}{1-p} \int_p^1 \left(\frac{s-p}{1-p}\right)^{n-1} \mathtt{VaR}_X(s)\,\mathrm{d}s.$$

This is the classical $\mathtt{ES}$ when $n = 1$.

Motivated by the work of Li and Wang [2] for $\mathtt{ES}$, we define the probability equivalent level of $\mathtt{VaR}$ and n^{th}-order $\mathtt{ES}$, called $\mathtt{PELVE}_n$. For an integrable random variable X and $\varepsilon \in (0,1)$, the $\mathtt{PELVE}_n$ of X at the level ε is the infimum of those values $c \in [1, \frac{1}{\varepsilon}]$ for which $\mathtt{ES}_{X,n}(1-c\varepsilon) \leqslant \mathtt{VaR}_X(1-\varepsilon)$. One can see that the level $\varepsilon = 0.01$ corresponds to the replacement of $\mathtt{VaR}$ at the level 0.99 with n^{th}-order $\mathtt{ES}$ at some appropriate level. We study the properties of $\mathtt{PELVE}_n$ and calculate $\mathtt{PELVE}_2$ for some important distributions including ones with heavy tail. Moreover, for $\mathtt{PELVE}_2$, we present some simulation results along with real data analysis.

[1] Barczy, M., K. Nedényi, F. and Sütő, L. (2022). Probability equivalent level of Value at Risk and higher-order Expected Shortfalls.
ArXiv: https://arxiv.org/abs/2202.09770

[2] Li, H. and Wang, R. (2019). PELVE: Probability Equivalent Level of VaR and ES. To appear in *Journal of Econometrics. Available also at SSRN.* **30(2)** 325–341.
URL: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3489566

On Group Algebras with Metabelian Group of Units

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Let F be a field of characteristic p, G a group and denote by $U(FG)$ the group of units of the group algebra FG. In 1991 Shalev [1] gave a necessary and sufficient condition for $U(FG)$ to be metabelian (by a metabelian group we mean a group with an abelian commutator subgroup), provided that $p > 2$ and G is finite. A few years later Kurdics [2] and, independently, Coleman and Sandling [3] did the same for the case $p = 2$. (It's important to note, that in theory of group rings the $p = 2$ case is nearly always exceptional and require a different treatment compared to the $p > 2$ case.) The restriction for the order of G was relaxed by Catino and Spinelli [4], such that G may be torsion for odd p, and nilpotent torsion for $p = 2$. Recently Juhász and Spinelli [5] have handled the non-torsion case for odd p.

In this work, we take some steps to this direction for the $p = 2$ case. In particular, we find all non-nilpotent groups G, for which $U(FG)$ is metabelian. We do not impose any restriction to the order of G, however, when G is not torsion, we must require the group algebra FG to be modular (that is, G contains an element of order 2). Namely, we prove the following theorems.

Theorem 1. *Let G be a non-abelian torsion group and F a field of characteristic 2. Then $U(FG)$ is metabelian, if and only if, one of the following statements holds:*

1. *The commutator subgroup G' of G is a central elementary abelian group of order 2 or 4;*
2. *F is the field of two elements, and G is an extension of an elementary abelian 3-group H by the group $\langle b\rangle$ of order 2 with $b^{-1}ab = a^{-1}$ for every $a \in H$.*

Theorem 2. *Let G be a non-torsion group and F a field of characteristic 2. If FG is modular and $U(FG)$ is metabelian, than G is nilpotent.*

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[1] A. Shalev, Meta-Abelian unit groups of group algebras are usually Abelian, *J. Pure Appl. Algebra* **72** (1991), 295–302.

[2] J. Kurdics, On group algebras with metabelian unit groups, *Period. Math. Hung.* **32** (1996), No. 1–2, 57–64.

[3] D.B.Coleman, R. Sandling, Mod 2 group algebras with metabelian unit groups, *J. Pure Appl. Algebra* **131** (1998), 25–36.

[4] F. Catino, E. Spinelli, On the derived length of the unit group of a group algebra, *J. Group Theory* **13** (2010), 577–588.

[5] T. Juhász, E. Spinelli, Group rings with metabelian unit groups, *J. Pure Appl. Algebra* **226** (2022).

Pathway to Numerical Simulations of Stripped-Envelope Supernovae

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Core-collapse supernovae (CCSNe) are among the most intensely studied transients of modern astrophysics due to their significant part in cosmic nucleosynthesis and large-scale distant measurements. Despite that CCSNe play a key role in many astrophysical aspects, we still do not understand some basic features of these objects. Moreover, the determination of the physical properties of SNe (e.g., explosion energy, ejected mass) from observational data is also quite uncertain. Recent high-cadence transient surveys have started to reveal some interesting phenomena of the supernova environment via following the light variation of these objects taken within a few days after the explosion. The observations taken at very early phases provide essential information on the radius of the progenitor star and the properties of the circumstellar matter (CSM). Examining the trace of the CSM could be especially important for stripped-envelope SNe, which lost most of its outer hydrogen and helium layers just before the supernova explosion. They need to get rid of their outermost layers during stellar evolution, but the exact mechanism and timescale of mass-loss processes are not known yet. Thus, numerical simulations are essential to understand better the complex physical nature of these exploding stars.

Moreover, in the past half-century, theoretical studies (e.g., [1], [2]) revealed a discrepancy in the derived ejecta masses from early- and late-time light curve (LC) fits of these objects. To solve this problem, we should consider two different scenarios. First, it is plausible that the mass discrepancy occurs due to the limitations and initial boundary conditions of our semi-analytic models. On the other hand, this mass discrepancy may have a physical cause, such as low-mass, low-density ejecta, or CSM around the supernova remnant. Here, I aim to present our recent studies, which may ease the tension between the estimated ejecta masses and get closer to solving the long-lasting mass-discrepancy problem of stripped-envelope SNe.

[1] A. Clocchiatti, J. C. Wheeler, On the Light Curves of Stripped-Envelope Supernovae, *Astrophysical Journal* **491** (1997), 375–380.

[2] J. C. Wheeler, V. Johnson, A. Clocchiatti, Analysis of late-time light curves of Type IIb, Ib and Ic supernovae, *Monthly Notices of the Royal Astronomical Society* **450** (2015), 1295–1307.

Best and Random Approximations with Generalized Disc–Polygons

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In this talk, we consider the asymptotic behaviour of the distance between a convex disc K with sufficiently smooth boundary, and its approximating n-gons, as the number of vertices tends to infinity. We consider two constructions: the best approximating inscribed n-gon of K is the one with maximal area; and a random inscribed n-gon of K is the convex hull of n i.i.d. random points chosen from the boundary of K. The asymptotic behaviour of the area deviation of K and the n-gon depend in both cases on the same, geometric limit. The best and random approximating n-gons can be similarly defined in the circumscribed case.

We generalize the existing results on linear and spindle convexity to the so-called L-convexity. In the case of inscribed L-polygons, we prove similar asymptotic formulae by generalizing the geometric limits. Then we introduce an L-convex duality, consider its properties, and use them to prove the formulae for the circumscribed cases.

This is joint work with Viktor Vígh (University of Szeged).

This research was supported by the ÚNKP-21-1 New National Excellence Program of the Ministry for Innovation and Technology from the source of the National Research, Development and Innovation Fund.

[1] F. Fodor, V. Vígh Disc-polygonal approximations of planar spindle convex sets, *Acta Sci. Math. (Szeged)* **78** (2012), no 1-2, 331-350

[2] Zs. Lángi, M. Naszódi, I. Talata Ball and spindle convexity with respect to a convex body, *Aequationes Math.*, **85** (2013) no 1-2, 41–67

[3] D. E. McClure, R. A. Vitale, Polygonal Approximation of Plane Convex Bodies, *J. of Math. Analy. and Appl.* **51** (1975), 326–358.

[4] R. Schneider, Random approximation of convex sets, *Journal of Microscopy*, **151** (1988), 211–227

Global Dynamics of a Within-Host Model for Usutu Virus

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In this work, we propose a within-host mathematical model for the Usutu virus with Crowley–Martin functional response. The basic reproduction number R0 is found by the next-generation matrix method. The model exhibits one or two equilibria, depending on the basic reproduction number. The local stability of the two equilibria is discussed using the Routz–Hurwitz criterion. Global stability is also established by constructing appropriate Lyapunov functions and using LaSalle's invariance principle. Numerical simulations are presented to illustrate the results.

[1] N. Heitzman-Breen, J. Golden, A. Vazquez, S. C. Kuchinsky, N. K. Duggal, S. M. Ciupe, Modeling the dynamics of Usutu virus infection of birds, *J. Theor. Biol.* **531** (2021), 110896.

[2] Rubel, F., Brugger, K., Hantel, M., Chvala-Mannsberger, S., Bakonyi, T., Weissenböck, H., Nowotny, N. (2008). Explaining Usutu virus dynamics in Austria: model development and calibration. *Preventive veterinary medicine,* **85(3-4)**, 166-186.

[3] Vargas-De-León, C. (2009). Constructions of Lyapunov functions for classic SIS, SIR and SIRS epidemic models with variable population size. Foro-Red-Mat: *Revista electrónica de contenido matemático*, **26**, 1-12.

On Semi-Symmetric Compatible Linear Connections in Finsler Geometry

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Finsler metrics are direct generalizations of Riemannian metrics such that the quadratic Riemannian indicatrices in the tangent spaces of a manifold are replaced by more general convex bodies as unit spheres. A linear connection on the base manifold is called compatible to the Finsler metric if the induced parallel transports preserve the Finslerian length of tangent vectors. The equations describing these connections are the so-called compatibility equations. Finsler manifolds admitting compatible linear connections are called generalized Berwald manifolds. Although there are some theoretical results for the solvability of the compatibility equations, the problem is very hard to solve in general because compatible linear connections may or may not exist on a Finsler manifold and may or may not be unique. Therefore special cases are of special interest.

The case we are concerned with here is the so-called semi-symmetric compatible linear connection, whose torsion tensor can be decomposed using a 1-form on the base manifold. It was proved by Vincze ([1]) that such a connection must be unique (if it exists) and the 1-form in the decomposition of the torsion tensor can be expressed by averaging, i.e. integration of differential forms on the tangent manifold over the Finslerian indicatrices. This is an elegant theoretical result, but the integrals obtained are hard to compute in practice.

Now, we are going to present a new proof of the unicity of semi-symmetric compatible linear connections, using only basic linear algebra and some elementary properties of convex bodies. This method yields an expression of the (only possible) solution easier to compute than the original integral formulas, and we also obtain intrinsic equations (without any unknown quantities) describing necessary conditions of the solvability.

This research was supported by the UNKP-21-3 New National Excellence Program of the Ministry for Innovation and Technology from the source of the National Research, Development and Innovation Fund of Hungary.

[1] Cs. Vincze, Generalized Berwald manifolds with semi-symmetric linear connections, *Publ. Math. Debrecen* **83** (4) (2013), pp. 741–755.

[2] Cs. Vincze, M. Oláh, Finsler metrics and semi-symmetric compatible linear connections, manuscript, arXiv:2204.01034

Interpretations of Nash Equilibrium Points

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Even though the Nash equilibrium is a concept of game theory, we apply it in economy, in geometry as well. The Nash equilibrium is an optimal outcome of a procedure where every player keeps their strategy, and does not want to deviate from their chosen. It conceptualizes the behavior of game participants to determine the best outcomes. It also contains predictions regarding the decision of the players if they use same decisions at the same time. In this study we compute Nash equilibrium points and visualize the output of the chosen algorithm.

There exist several algorithms to compute Nash equilibrium points, such as Lemke and Howson, Elzen and Talman (see [1]), Harsanyi (see [2]), etc. One of the most popular implementation is the Lemke and Howson algorithm which was developed for bimatrix games. This algorithm is "the best known among the combinatorial algorithms for finding a Nash equilibrium" where each players' game is interpreted as a best-response polytope. The Lemke-Howson algorithm uses the idea of maintaining a single guess as to what the supports should be, and in each iteration the guess is changed only a little bit. In this study we are going to present the mathematical concept of Nash equilibrium points and a data visualization based on the Lemke and Howson algorithm. The aim of visualization dashboard is to expand our limited study for uncountable cases with freely adjustable input. This tool makes the Nash equilibrium points more understandable for general users independently of the context of the use of the Nash equilibrium points.

The aim of data visualization tool is to become more distant from the game theory concept of the Nash equilibrium points and based on mathematical functions to determine a more general form of this this type of equilibrium point. This study also contains a summery of existing algorithms.

[1] A. H. Elzen and A. J. J. Talman. A procedure for finding nash equilibria in bimatrix games. *ZOR Zeitschrift für Operations Research Methods and Models of Operations Research*, 35(1):27–43, January 1991.

[2] J. C. Harsanyi. The tracing procedure: A bayesian approach to defining a solution forn-person noncooperative games. *International Journal of Game Theory*, 4(2):61–94, June 1975.

Nonconstant Periodic Solutions of a Nonlinear Delay Equation

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The delay differential equation

$$x'(t) = -ax(t) + bf\left(x(t-1)\right) \tag{E}$$

is considered where $a > 0,\ b > 0$ and a continuously differentiable function $f : \mathbb{R} \to \mathbb{R}$ satisfying $f(0) = 0,\ f'(\xi) > 0$ for $\xi \in \mathbb{R}$. It is well-know that if 0 is hyperbolic then it has a neighborhood in which there exists no nontrivial periodic orbit. By using the exponential dichotomy constants, we focus on the estimation of the optimal size of this neighborhood. The aim is to construct the neighborhood as large as possible in order to be able to carry out a verified numerical step for equation (E).

[1] O. Diekmann, et al., *Delay Equations: Functional-, Complex-, and Nonlinear Analysis*, Applied Mathematical Sciences, Vol. 110, Springer-Verlag, New York, 1995.

[2] A. B. Ferenc, T. Krisztin, A. Vígh, Stable periodic orbits for the Mackey–Glass equation, *J. Differ. Equ.* **296** (2021), 15–49.

[3] J. Hale, S. M. V. Lunel, *Introduction to Functional Differential Equations*, Vol. 99, Springer-Verlag, New York, 1993.

[4] T. Krisztin, H. O. Walther, J. Wu, *Shape, Smoothness and Invariant Stratification of an Attracting Set for Delayed Monotone Positive Feedback*, American Mathematical Society, Vol. 11, Rhode Island, 1999.

[5] T. Krisztin, Periodic solutions with long period for the Mackey–Glass equation, *Electron. J. Qual. Theory Differ. Equ.* **83** (2020), 1–12.

Epidemic Patterns of Emerging Variants

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Motivated by the emergence of new variants during the COVID-19 pandemic, we consider an epidemiological model of disease transmission dynamics, where by mutation novel strains of a virus appear. In the model, disease prevalence is modulated by social distancing. We study the different patterns that are generated under different assumptions. If recovery from a given strain gives immunity against all previous strains, but not against more novel strains, then we observe a very regular sequential pattern of strain replacement where newer strains gain dominance, and their waves are increasingly wider. However, if protection upon recovery holds only against that particular strain, we find a much more complicated dynamics with potential recurrence of earlier strains, and co-circulation of various strains. Finally, we compare the observed patterns with what we have seen during COVID-19.

[1] Lobinska G, Pauzner A, Traulsen A, Pilpel Y, Nowak MA. Evolution of resistance to COVID-19 vaccination with dynamic social distancing. Nat Hum Behav. 2022;6:193–206.

[2] Massard, Mathilde and Eftimie, Raluca and Perasso, Antoine and Saussereau, Bruno, *A multi-strain epidemic model for COVID-19 with infected and asymptomatic cases: Application to French data*, Journal of Theoretical Biology, Elsevier, 2022.

On the Variance of the Volume of Random Polytopes

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We study the geometric properties of random polytopes that arise as the convex hull of n i.i.d. random points selected from a fixed convex body K in d-dimensional Euclidean space. In particular, we are interested in the variance of the volume of such random polytopes. Since the seminal works of Rényi and Sulanke (1963, 1964, 1968) a large part of results on random polytopes are asymptotic in nature, that is, valid under the assumption that $n \to \infty$. In this spirit, we are interested in upper and lower estimates of the order of magnituded of the variance of the volume of random polytopes in terms of n as $n \to \infty$. We consider some probability models in which such estimates are still missing.

Stochastic Modeling of the Bactericidal Potency

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We develop a stochastic model for bacterial growth, where the size of the bacterial population follows a Bienaymé–Galton–Watson branching process, and the offspring distribution depends on the concentration of the antibiotic.

For a fixed antibiotic concentration $c > 0$ we provide an estimator for the offspring mean $m(c)$, and show that the estimator is weakly consistent and asymptotically normal. We provide a consistent estimator for the minimal inhibitory concentration (MIC), which is the smallest concentration preventing bacterial growth, an important parameter in microbiology.

As a real data we used measurements of *Chlamydia trachomatis* growth which was analyzed by a novel quantitative PCR method treated by 4 different antibiotics at 12 different antibiotic concentrations. We found that our model fits very well to this data.

Higher Order Homological Percolation on the Torus

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In percolation theory usually one asks questions about the appearance of large components with given properties in a random space. In [1] Duncan, Kahl and Schweinhart considered a so callad plaquette percolation - very similar to Bernoulli percolation - on the torus and proved results on the critical values of different properties of the image of the homology group induced by the natural inclusion. They proved that some critical values in the middle dimension are 1/2 but for other dimensions the concrete values are unknown. I will talk about about this percolation model, present some of my approaches to the problem and simulation results.

[1] Duncan, P., Kahle, M., and Schweinhart, B., Homological percolation on a torus: plaquettes and permutohedra, *arXiv preprint arXiv:2011.11903.* (2020).

Teaching the Basics of Artificial Intelligence at High School

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Attila József Secondary Grammar School, Makó, Hungary

Since data science or artificial intelligence (AI) related courses will appear in all STEM curricula, it is very important for the student to meet with the basics at high school. In the collaboration of Attila József Secondary Grammar School in Makó and the Bolyai Institute, we started implementing an extracurricular program, where we discuss the foundations of machine learning with students from Mathematics-Informatics specialization. The approach is interdisciplinary focusing on mathematical aspects of the topics. The curriculum consists of two 18-hour modules mixing the mathematical and the programming elements.

In this talk I will introduce the content of the two modules and the applied methodologies. I will compare the experiences from the sessions at the test phase of the program (from 2019-2020) and the officially held first round (from 2021-2022). I will present the didactic and content changes, which we have to elaborate during the first round and I will share my opinion about how to incorporate similar materials into the local curricula and how these changes could fortify the bridge between the secondary and the higher education. This is a joint work with Norbert Bogya and Zsolt Vizi from Bolyai Institute, University of Szeged.

The presented curriculum and methodologies are parts of my ongoing Master Educator (Mestertanári) Program.

Interdisciplinary Teaching of Throws in High School

Tekeli Miklós, János Karsai
University of Szeged, Szeged, Hungary

During my years as a high school teacher I often observed that in high school they teach mathematics completely separately from other science subjects. Those who do not experience the beauty of mathematics can find the topics of study a bit arbitrary without real-life applications. Students tend to be interested in the laws of science which describe our world, so it's easy to see they could like mathematics more through physics. From my experiences, the visualization of the observed phenomena through various softwares can enlight the topic in a different way. Therefore, I think that modelling through softwares play a huge role in teaching science in high school. During my doctoral studies, I will be working on a textbook intended as a methodological guide which will mostly contain modelling tasks broken down by grade and subject for intermediate and advanced level groups. Through the course of model selection I will mainly take into account having the concepts of differential and integral calculus be related to the concepts of physical sciences in the projects.

In this talk I will show how one can teach the concept of projectile motion with the help of mathematics, physics and visualization with softwares. One can easily do experiments with this phenomenon but observing and describing this with mathematical tools can be interpreted in different ways.

[1] M. Tekeli, Interdisciplinary teaching of throws in high school. (in Hungarian), *Bolyai Institute, University of Szeged* (2020)

Adaptive Group Testing in a COVID-19 Model

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Various measures have been implemented around the world to prevent the spread of SARS-CoV-2. A potential tool to reduce disease transmission is regular mass testing of a high percentage of the population, possibly with pooling (testing a compound of several samples with one single test). We develop a compartmental model to study the applicability of this method and compare different pooling strategies: regular and Dorfman pooling. The model includes isolated compartments as well, from where individuals rejoin the active population after some time delay. We develop a method to optimize Dorfman pooling depending on disease prevalence and establish an adaptive strategy to select variable pool sizes during the course of the epidemic. It is shown that optimizing the pool size can avert a significant number of infections. The adaptive strategy is much more efficient, and may prevent an epidemic outbreak even in situations when a fixed pool size strategy can not.

[1] P. Boldog, T. Tekeli, Z. Vizi, A. Dénes, F. A. Bartha, G. Röst, Risk assessment of novel coronavirus COVID-19 outbreaks outside China, *J. Clin. Med*, **9** (2020), 571. https://doi.org/10.3390/jcm9020571

[2] H. Nishiura, S.-M. Jung, N. M. Linton, R. Kinoshita, Y. Yang, K. Hayashi, et al., The extent of transmission of novel coronavirus in Wuhan, China, *J. Clin. Med.*, **9** (2020), 330. https://doi.org/10.3390/jcm9020330

Using Permutation Games in the Teaching of Functions

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Function is a commonly used concept in mathematics, and its teaching often poses serious challenges for teachers. In my presentation, I am answering the question of whether permutation games can make it easier for primary and secondary school students to understand the concept of function. I will cover some permutation games in detail and their mathematical background as well. I will briefly describe the history of the concept of functions and its appearance and development in school textbooks. Then I will present my own research about using permutation games in the teaching of functions.

During my research, one of my goals was to try out some permutation games with students to find out how understandable this topic (which is only taught at the university level) is for a student in public education. Furthermore, if it is understandable, how easy it is for them to acquire and use this knowledge. I believe that this playful approach helps to overcome math phobia, and, through permutations, students can see another approach to interpreting functions, perhaps making it easier to introduce functions defined on infinite sets.

To answer my questions, I presented the 15 puzzle and the condition of its solvability in 14 classes. I tried to make a good atmosphere during the classes and made time for explorations and listening to students' ideas to observe how much they liked the topic. Besides having fun in math class, effectiveness was important, too. At the end of the lessons, each student had to complete a test. The result and students' reactions were encouraging: A about 80% of the students were able to write the starting position as a product of cycles.

After the 15 puzzle, I played three different swapping games with 3 classes. I wanted to transfer the knowledge students gained during these games to functions, however, I have not had enough time for that yet. The experience so far is very promising, so it is worth pursuing this topic further. Students are interested in different games and are happy to play with them. Moreover, based on the results, it seems possible with serious preparations to make the essence of functions understandable with the help of games.

[1] Z. Dienes, *Építsük fel a matematikát*, SHL Hungary Kft., Budapest, 1999.

[2] A. Sfard, On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin, *Educational Studies in Mathematics* **22** (1991), 1–36.

[3] D. Tall, S. Vinner, Concept images and concept definition in mathematics with particular reference to limits and continuity, *Educational Studies in Mathematics* **12** (1981), 151–169.

Teaching Mathematics During the COVID-19 Lockdown

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In the first half of the presentation, I plan to illustrate how three mathematics teachers, university teachers and students (Dr. Attila Máder, Bence Torma, Gábor Torma) with different levels of experience but the same enthusiasm and willingness to experiment, experienced the digital work under the lockdown through different tools and platforms.

I will highlight how the difficulties we faced motivated us to look for new ways, tools, methods, and also how the critical examination of these influenced and shaped the methods we used during lessons and to monitor knowledge (e.g., the use of gamification). In addition, I will point out the difficulties of the first switch, the problems that arose when choosing educational platforms, the possibilities of holding lessons, the development of teaching materials and the changing role of evaluation.

In the second half of the presentation, I will show in light of the answers to a questionnaire, how students look back on the situation they have experienced, and what lessons could be drawn from their answers about the previous and current state of mathematics education.

I hope that after the presentation, more people will be encouraged to use the mentioned or similar tools to expand their methodological toolbox, which can be useful not only in the next similar periods, but also in the successful teaching of the alpha-generation.

[1] A. Máder, B. Torma, G. Torma, Matematikaoktatás a COVID-19 idején, *Érintő - Elektronikus Matematikai Lapok* **18** (2020)

[2] R. Fromann, A. Damsa, A gamifikáció (játékosítás) motivációs eszköztára az oktatásban, *Új Pedagógiai Szemle* **66** (2016), 76-81.

[3] *PISA 2018. Összefoglaló jelentés, www.oktatas.hu*

Right Regular Triples of Semigroups

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In the field of abstract algebra, matrices are being researched extensively. The applications of matrices have been present in many other fields of mathematics as well, hence its a very popular topic. There are two types of matrices in the theory of semigroups which play an important role in the investigation of the structure of semigroups. They are the Rees matrices over a semigroup [3] (especially, over a group with a zero adjoined) and the right matrices of elements of a semigroup. The right matrices of elements of finite semigroups are studied in [1]. In the present paper, a connection is made between these two matrix notions.
Let $\mathcal{M}(S;\Lambda;P)$ denote a Rees $I\times\Lambda$ matrix semigroup without zero over the semigroup S, where I is a singleton, and so P is a mapping of Λ into S. Let θ_S denote the kernel of the right regular matrix representation of a semigroup S. In [2, Theorem 1], it is proved that if S is an arbitrary semigroup and P is a choice function on the collection of all θ_S-classes of S, then the factor semigroup $\mathcal{M}(S;S/\theta;P)/\theta_{\mathcal{M}(S;S/\theta;P)}$ is isomorphic to the Rees matrix semigroup $\mathcal{M}(S/\theta^*;S/\theta;P')$, where θ_S^* is the right colon congruence of θ_S, and P' is the mapping of S/θ_S onto S/θ_S^* such that $P'([s]_{\theta_S})=[s]_{\theta_S^*}$ for every $s\in S$. In other words, the semigroup of all right matrices of the Rees matrix semigroup $\mathcal{M}(S;S/\theta_S;P)$ is isomorphic to the Rees matrix semigroup $\mathcal{M}(S/\theta^*;S/\theta;P')$. On the base of this result, the following property of a sequence of semigroups is defined. A triple A,B,C of semigroups is said to be right regular, if there are mappings

$$A \xleftarrow{P} B \xrightarrow{P'} C$$

such that

$$\mathcal{M}(A;B;P)/\theta_{\mathcal{M}(A;B;P)} \cong \mathcal{M}(C;B;P').$$

In our present paper we focus on right regular triples of semigroups. By [2, Theorem 1], if A,B,C are semigroups such that $A/\theta_A\cong B$ and $B/\theta_B\cong C$, then the triple A,B,C is right regular. In [2], a right regular triple A,B,C is also given, where none of $A/\theta_A\cong B$ and $B/\theta_B\cong C$ is fulfilled. These examples motivate us to construct right regular triples of semigroups, and examine the connection between the semigroup theoretical properties of semigroups belonging to a right regular triple of semigroups.

[1] A. Nagy, Cs. Tóth, On the probability that two elements of a finite semigroup have the same right matrix, *Comment.Math.Univ.Carolin.* (to appear)

[2] A. Nagy, Cs. Tóth, On special Rees matrix semigroups over semigroups *https://doi.org/10.48550/arXiv.1609.09821 (preprint)*

[3] D. Rees, On semi-groups, *Proc. Cambridge Philosophical Society,* **36** (1940), 387-400.

Polymorphism-Homogeneous Groupoids on the Three-Element Set

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Previously we proved that if we assign a certain relational structure to any finite algebra in a canonical way, using solution sets of equations, then this relational structure is polymorphism-homogeneous if and only if the algebra itself is polymorphism-homogeneous [1]. Furthermore, we showed that polymorphism-homogeneity is also equivalent to the property that algebraic sets (i.e., solution sets of systems of equations) are exactly those sets of tuples that are closed under the centralizer clone of the algebra. In addition to this, we proved that the aforementioned properties hold if and only if the algebra is injective in the category of its finite subpowers. Therefore, there are several possibilities for investigating polymorphism-homogeneity. Our current goal is to determine which three-element groupoids are polymorphism-homogeneous. There are 19683 groupoids on the three-element set, but up to isomorphism their number is only 3330. In the article [2] Joel Berman and Stanley Burris investigate multiple properties of these groupoids with the help of computers. We continue this path by using a computer to help us make it clearer which groupoids are polymorphism-homogeneous. Naturally, some conjectures arise from the result given by our program(s). We focus on these conjectures, and would like to give some general condition for a groupoid to be polymorphism-homogeneous.

[1] E. Tóth and T. Waldhauser, *Polymorphism-homogeneity and universal algebraic geometry,* Discrete Mathematics & Theoretical Computer Science, special issue in honour of Maurice Pouzet **23**:(2), 2022

[2] J. Berman and S. Burris, *A computer study of 3-element groupoids,* In Logic and Algebra, (The Proceedings of the Magari Conference), 379 – 430, Marcel Dekker, Inc, 1996

Ulam-Hyers Stability of Linear Differential Systems on Unbounded Intervals

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In this paper, first we give an alternate proof for the characterization of the Ulam-Hyers stability of the differential system

$$x' = A(t)x, \quad t \in \mathbb{R}, \quad x(t) \in X^n,$$

with periodic real matrix $A(t)$, where X is a Banach space. That is, it is Ulam-Hyers stable, if and only if the monodromy matrix associated to the family $\{A(t)\}_{t\geq 0}$ has no eigenvalues on the unit circle in $\mathbb{C}$. For the case when only $\mathbb{R}_+$ is considered the authors Barbu, Buşe, and Tabassum have given the same characterization in their paper [1] in 2015, where the matrix $A(t)$ is a complex matrix and $X = \mathbb{C}$.

Second we will show that the higher order differential equation

$$x^{(n)} + a_1(t)x^{(n-1)} + \ldots + a_n(t)x = 0, \quad t \in \mathbb{R}, \quad x(t) \in X^n,$$

with T-periodic real coefficients, is Ulam-Hyers stable if and only if the associated differential system

$$y' = A(t)y,$$

is Ulam-Hyers stable, where $y = (x, x', \ldots, x^{(n-1)})$. In the case of constant coefficients it was shown by the authors in 2003 [2], that the equation is Ulam-Hyers stable exactly when the characteristic polynomial has no pure imaginary roots.

These are joint results with Assoc. Prof. dr. Adriana Buică, from Babeş-Bolyai University, Faculty of Mathematics and Computer Science, Romania.

[1] Constantin Buse, Dorel Barbu, and Afshan Tabassum. Hyers-Ulam stability and exponential dichotomy of linear differential periodic systems are equivalent. *Electronic Journal of Qualitative Theory of Differential Equations*, **2015** (2015), 1–12.

[2] Takeshi Miura, Shizuro Miyajima, and Sin-Ei Takahasi. Hyers-Ulam stability of linear differential operator with constant coefficients. *Mathematische Nachrichten*, **258** (2003), 90–96.

Dynamical Behaviour of a Population-Based Neuron Model with Adaptation Current

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Mathematical modelling of neuronal networks plays an important role in understanding how human brain and memory work. One of the most widely used types of neuron models is the rate model, where the state variable represents the firing rate of a neuronal population in the system of differential equations. These models can be treated as dynamical systems enabling us to use bifurcation theory to investigate the changes in the states of neurons.

We study the behaviour of a deterministic rate model which describes the dynamics of an excitatory and inhibitory neuron population. Adaptation current is also added to the excitatory cells as a negative feedback. Our goal is to reproduce the behaviour of neurons that neurobiologists observe in their experiments, including different types of oscillations. We focus on the effect of adaptation and the self weight of the excitatory population. We detect the saddle-node bifurcation curve in the system which shows where the number of steady states changes. Stability of equilibria and appearance of periodic solutions are also studied by determining Andronov-Hopf bifurcation in the rate model. Both of these bifurcations are local bifurcations where the phase portrait changes in a neighbourhood of an equilibrium and they can be investigated analytically. Applying ReLU activation function allows us to give explicit formulas for the bifurcation curves. We show a few examples for the more interesting phase portraits. We identify the detected periodic orbits with different types of oscillations observed in the brain.

[1] A. Ecker, et al. Hippocampal sharp wave-ripples and the associated sequence replay emerge from structured synaptic interactions in a network model of area CA3. *Elife* **11** (2022) e71850.

[2] D. Levenstein, et al., NREM sleep in the rodent neocortex and hippocampus reflects excitable dynamics. *Nat communications* **10**(1) (2019), 1-12.

Notes

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TERMÉSZETTUDOMÁNYI ÉS INFORMATIKAI KAR

MAGYAR TUDOMÁNYOS AKADÉMIA · 1825 ·

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