Asymptotic inference for linear stochastic differential equations with distributed time delay

János Marcell Benke, Gyula Pap
University of Szeged, Szeged, Hungary

Assume that we observe a stochastic process $(X(t))_{t\in[-r,T]}$, which satisfies the linear stochastic delay differential equation

$$\mathrm{d}X(t) = \vartheta \int_{[-r,0]} X(t+u)\, a(\mathrm{d}u)\, \mathrm{d}t + \mathrm{d}W(t), \qquad t \geq 0,$$

where a is a finite signed measure on $[-r, 0]$. The local asymptotic properties of the likelihood function are studied.

In the first part of the talk, we study a special case, when the delay is uniform. Namely, when a is the Lebesgue-measure and $r = 1$. In this special model local asymptotic normality is proved in case of $\vartheta \in \left(-\frac{\pi^2}{2}, 0\right)$, local asymptotic mixed normality is shown if $\vartheta \in \left(0, \infty\right)$, periodic local asymptotic mixed normality is valid if $\vartheta \in \left(-\infty, -\frac{\pi^2}{2}\right)$, and only local asymptotic quadraticity holds at the points $-\frac{\pi^2}{2}$ and 0.

In the general model we have not chance to determinate the exact values of the parameter, where the appropriate property is valid. However we can add a sufficient condition for this. Local asymptotic normality is proved in case of $v_\vartheta^* < 0$, local asymptotic quadraticity is shown if $v_\vartheta^* = 0$, and, under some additional conditions, local asymptotic mixed normality or periodic local asymptotic mixed normality is valid if $v_\vartheta^* > 0$, where v_ϑ^* is an appropriately defined quantity. As an application, the asymptotic behaviour of the maximum likelihood estimator $\widehat{\vartheta}_T$ of ϑ based on $(X(t))_{t\in[-r,T]}$ can be derived as $T \to \infty$.

[1] J. M. Benke, G. Pap, Asymptotic inference for a stochastic differential equation with uniformly distributed time delay, *Journal of Statistical Planning and Inference* **167** (2015), 182–192.

[2] J. M. Benke, G. Pap, One-parameter statistical model for linear stochastic differential equation with time delay, submitted to *Statistics*. Available on arXiv: http://arxiv.org/abs/1510.04115