Parameter estimation for subcritical Heston models based on discrete time observations

Mátyás Barczy, Gyula Pap, **Tamás T. Szabó**
University of Szeged, Szeged, Hungary

Heston models have been extensively used in financial mathematics since one can well-fit them to real financial data set, and they are well-tractable from the point of view of computability as well. Hence parameter estimation for Heston models is an important task.

In the talk we study conditional least squares estimators (CLSEs) and least squares estimators (LSEs) for Heston models

$$\begin{cases} dY_t = (a - bY_t)\,dt + \sigma_1\sqrt{Y_t}\,dW_t, \\ dX_t = (\alpha - \beta Y_t)\,dt + \sigma_2\sqrt{Y_t}\big(\varrho\,dW_t + \sqrt{1-\varrho^2}\,dB_t\big), \end{cases} \qquad t \geq 0, \tag{1}$$

where $a > 0$, $b, \alpha, \beta \in \mathbb{R}$, $\sigma_1 > 0$, $\sigma_2 > 0$, $\varrho \in (-1, 1)$, and $(W_t, B_t)_{t\geq 0}$ is a 2-dimensional standard Wiener process. We investigate only the so-called subcritical case, i.e., when $b > 0$. It is well-known that in this case the process $(Y_t)_{t\geq 0}$, which is just the Cox–Ingersoll–Ross process, is ergodic. We consider a CLSE and LSE of (a, b, α, β) based on discrete time observations of the process $(X_t, Y_t)_{t\geq 0}$, when the parameters σ_1, σ_2 and ϱ are assumed to be known.

We use the method of conditional least squares, which was first applied to the CIR process by Overbeck and Rydén [1]. We estimate a suitably transformed parameter vector (c, d, γ, δ), for which the estimation error can be written as a sum of martingale differences. The strong consistency and the asymptotic normality follow from this fact using the strong law of large numbers and the central limit theorem for square-integrable martingales. The asymptotic covariance matrix is derived for the estimation errors of (c, d, γ, δ) as well as the estimation errors of the original parameters (a, b, α, β).

We also introduce a plausible set of estimators based on the ordinary least squares method, show that they are not consistent, and we derive their strong limit.

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[1] L. Overbeck, T. Rydén., Estimation in the Cox-Ingersoll-Ross model, *Econometric Theory* **13** (1997), 430–461.