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Time reversible attractors of parabolic PDEs: A meandering tale of three noses

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As a paradigm of irreversibility, we consider dissipative parabolic semilinear "heat equations"

$$u_t = u_{xx} + f(x, u, u_x) \tag{PDE}$$

on the unit interval with Neumann boundary. By results of Angenent, Brunovský, Fusco, Henry, Matano, Rocha, Wolfrum, and myself, among others, the global attractors are determined by the shooting meanders for the ODE boundary value problem $0 = v'' + f(x, v, v')$. The celebrated 1974 Chafee-Infante case of cubic f is characterized by meanders with two noses.

As a natural next step, we investigate the heteroclinic connection graphs for meanders with three noses, i.e. innermost arcs to p, q, and $p + q$ nested meander arcs, respectively. Such meanders arise in the PDE context if, and only if, $p = r(q + 1)$, for some integer r. Surprisingly, all their connection graphs are *time reversible*: a rather non-intuitive bijection of equilibria *reverses* all heteroclinic directions.

We also explore the case $q = r(p - 1)$, which leads to the absurdity of negative "unstable dimensions" ranging from $1 - r$ to -1. Only after $r - 1$ unstable double cone suspensions, such meanders first provide parabolic attractors again. Much to our surprise, their connection graphs then seem to coincide with their time-reversible cousins above.

For general 3-nose meanders however, i.e. for general co-prime $p - 1$ and $q + 1$, a full classification remains elusive – after almost 50 years.

All this is joint work with Carlos Rocha.