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Oscillation and asymptotic properties of second order delay equations: New trends

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Delays, arising in nonoscillatory and stable ordinary differential equations, can induce oscillation and instability of their solutions. That is why the traditional direction in the study of nonoscillation and stability of delay equations is to establish a smallness of delay, allowing delay differential equations to preserve these convenient properties of ordinary differential equations with the same coefficients. In this talk, we find cases in which delays, arising in oscillatory and asymptotically unstable ordinary differential equations, induce nonoscillation and stability of delay equations. We demonstrate that, although the ordinary differential equation $x''(t) + c(t)x(t) = 0$ can be oscillating and asymptoticaly unstable, the delay equation $x''(t) + a(t)x(t - \tau(t) - b(t)x(t - \theta(t)) = 0$, where $c(t) = a(t) - b(t)$, can be nonoscillating and exponentially stable. Results on nonoscillation and exponential stability of delay differential equations are obtained. On the basis of these results on nonoscillation and stability, the new possibilities of non-invasive (non-evasive) control, which allow us to stabilize a motion of single mass point, are proposed. Stabilization of this sort, according to common belief requires damping term in the second order differential equation. Results obtained in this paper refutes this delusion.

It is demonstrated that, although the solutions of the delay differential equation

$$x^{''}(t) + \sum_{i=1}^{m} p_i(t)x(t - \tau_i(t)) = 0 \text{ for } t \in [0, \infty), \quad \text{where } x(\xi) = \varphi(\xi) \text{ for } \xi < 0,$$

considered for the zero intitial functions $\varphi(\xi) = 0$ can be oscillating with amplitudes tending to infinity, there can exist such initial functions $\varphi(\xi)$ that amplitudes of its oscillation solutions "started" with such φ tend to zero when $t \to \infty$. The fact of tending solutions to zero on the semiaxis is obtained through their "fast" oscillation, i.e. lenght of distance between adjacent zeros. The basis of this property is in the folowing: small distance between zeros does not allow amplitudes of oscillating solutions to increase. Even more, these amplitudes can tend to zero when $t \to \infty$. Results of this sort were considered as impossible. The exact estimates of this distances between zeros are proposed through estimates of the spectral radii of corresponding compact operators.