Unbounded solutions for differential equations
with mean curvature operator

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This is the joint work with Mauro Marini and Serena Matucci (University of Florence).

We present necessary and sufficient conditions for the existence of unbounded positive solutions for nonlinear differential equations

$$(a(t)\Phi_E(x'))' + b(t)F(x) = 0, \quad t \geq t_0,$$

where Φ_E denotes the Euclidean mean curvature operator

$$\Phi_E(u) = \frac{u}{\sqrt{1+u^2}}\,.$$

We use a fixed point result for operators defined in a Fréchet space, which does not require the explicit form of the fixed point operator and reduces the solvability of a boundary value problem for nonlinear equations to the solvability of an associated boundary value problem for a linear equation.

The results illustrate the asymptotic proximity of such solutions with those of an auxiliary linear equation on the threshold of oscillation. A new oscillation criterion for equations with mean curvature operator, extending Leighton criterion for linear Sturm–Liouville equation, is also derived.